1.(1 pt) Compute the 10th derivative of

$$f(x) = \arctan\left(\frac{x^2}{7}\right)$$

at $x = 0$.
$f^{(10)}(0) =$ ______
Hint: Use the MacLaurin series for $f(x)$.

2.(1 pt) Compute the 9th derivative of

$$f(x) = \frac{\cos(3x^2) - 1}{x^3}$$

at $x = 0$.
$f^{(9)}(0) =$ ______
Hint: Use the MacLaurin series for $f(x)$.

3.(1 pt) Find the degree 3 Taylor polynomial $T_3(x)$ of function

$$f(x) = (7x-38)^{3/2}$$

at $a = 6$.
$T_3(x) =$ ______

4.(1 pt) The Taylor series for $f(x) = x^3$ at -1 is $\sum_{n=0}^{\infty} c_n(x+1)^n$.
Find the first few coefficients.
$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______

5.(1 pt) The Taylor series for $f(x) = \ln(\sec(x))$ at $a = 0$ is $\sum_{n=0}^{\infty} c_n(x)^n$.
Find the first few coefficients.
$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______
Find the exact error in approximating $\ln(\sec(0.3))$ by its fourth degree Taylor polynomial at $a = 0$.
The error is ______

6.(1 pt) The Taylor series for $f(x) = \ln(\sec(x))$ at $a = 0$ is $\sum_{n=0}^{\infty} c_n(x)^n$.
Find the first few coefficients.
$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______

7.(1 pt) Find $T_4(x)$: the Taylor polynomial of degree 4 of the function $f(x) = \arctan(5x)$ at $a = 0$.
(You need to enter a function.)
$T_4(x) =$ ______

8.(1 pt) Find $T_{13}(x)$: the Taylor polynomial of degree 13 of the function $f(x) = \arctan(x^4)$ at $a = 0$.
(You need to enter a function.)
$T_{13}(x) =$ ______

9.(1 pt) Let $T_5(x)$ be the fifth degree Taylor polynomial of the function $f(x) = \cos(0.3x)$ at $a = 0$.
A. Find $T_5(x)$. (Enter a function.)
$T_5(x) =$ ______
B. Find the largest integer k such that for all x for which $|x| < 1$ the Taylor polynomial $T_5(x)$ approximates $f(x)$ with error less than $\frac{1}{10^k}$.
$k =$ ______

10.(1 pt) Match each of the Maclaurin series with right function.

— 1. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$
— 2. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
— 3. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
— 4. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

A. $\arctan(x)$
B. $\cos(x)$
C. $\sin(x)$
D. e^x

11.(1 pt) Match each of the Maclaurin series with right function.

— 1. $\sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+1}}{(2n+1)!}$
— 2. $\sum_{n=0}^{\infty} \frac{(-1)^n 2x^{2n+1}}{2n+1}$
— 3. $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$
— 4. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$

A. $2\sin(x)$
B. $\cos(2x)$
C. $2\arctan(x)$
D. e^{2x}

12.(1 pt) Select the FIRST correct reason why the given series diverges.

A. sin(x)

B. exp(x)
C. cos(x)
D. arctan(x)

— 1. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

— 2. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

— 3. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

— 4. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

13.(1 pt) Select the FIRST correct reason why the given series diverges.

A. sin(2x)
B. exp(2x)
C. cos(2x)
D. arctan(2x)

— 1. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$

— 2. $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

— 3. $\sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$

— 4. $\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{2n+1}$

14.(1 pt) Let $F(x) = \int_0^x \sin(4t^2)\, dt$.
Find the MacLaurin polynomial of degree 7 for $F(x)$.

Use this polynomial to estimate the value of $\int_0^{0.72} \sin(4x^2)\, dx$.

15.(1 pt) Let $F(x) = \int_0^x e^{-4t^4}\, dt$.
Find the MacLaurin polynomial of degree 5 for $F(x)$.

Use this polynomial to estimate the value of $\int_0^{0.19} e^{-4x^4}\, dx$.

16.(1 pt) Find the Maclaurin series of the function $f(x) = 8x^3 - 3x^2 - 7x + 3$
$(f(x) = \sum_{n=0}^{\infty} c_n x^n)$

$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______

Find the radius of convergence $R =$ ______ Enter INF if the radius of covergence is infinity .

17.(1 pt) Represent the function $x^{0.6}$ as a power series $\sum_{n=0}^{\infty} c_n (x-6)^n$.

$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______

Find the left endpoint of the interval of convergence.
left end = ______ .
Find the right endpoint of the interval of convergence.
right end = ______ .

18.(1 pt) Find Taylor series of function $f(x) = \ln(x)$ at $a = 9$.
$(f(x) = \sum_{n=0}^{\infty} c_n (x-9)^n)$

$c_0 =$ ______
$c_1 =$ ______
$c_2 =$ ______
$c_3 =$ ______
$c_4 =$ ______

Find the interval of convergence.
The series is convergent:
from $x =$ ______ , left end included (Y,N): ______
to $x =$ ______ , right end included (Y,N): ______

19.(1 pt) Evaluate

$$\lim_{x\to 0} \frac{\ln(1-x) + x + \frac{x^2}{2}}{12x^3}$$

Hint: Using power series.

20.(1 pt) Evaluate

$$\lim_{x\to 0} \frac{e^{-3x^3} - 1 + 3x^3 - \frac{9}{2}x^6}{12x^9}$$

Hint: Using power series.

21.(1 pt) Assume that $\sin(x)$ equals its Maclaurin series for all x.
Use the Maclaurin series for $\sin(5x^2)$ to evaluate the integral

$$\int_0^{0.75} \sin(5x^2)\, dx$$

.

Your answer will be an infinite series. Use the first two terms to estimate its value.

22.(1 pt) Assume that e^x equals its Maclaurin series for all x.
Use the Maclaurin series for e^{-5x^4} to evaluate the integral

$$\int_0^{0.11} e^{-5x^4}\, dx$$

Your answer will be an infinite series. Use the first two terms to estimate its value.

23.(1 pt) The Taylor series for $f(x) = e^x$ at a = -4 is $\sum_{n=0}^{\infty} c_n (x+4)^n$.

Find the first few coefficients.

$c_0 =$ ____

$c_1 =$ ____

$c_2 =$ ____

$c_3 =$ ____

$c_4 =$ ____

24.(1 pt) The Taylor series for $f(x) = \sin(x)$ at a = $\frac{\pi}{2}$ is $\sum_{n=0}^{\infty} c_n (x - \frac{\pi}{2})^n$.

Find the first few coefficients.

$c_0 =$ ____

$c_1 =$ ____

$c_2 =$ ____

$c_3 =$ ____

$c_4 =$ ____

25.(1 pt) The Taylor series for $f(x) = \sin(x)$ at a = $\frac{\pi}{3}$ is $\sum_{n=0}^{\infty} c_n (x - \frac{\pi}{3})^n$.

Find the first few coefficients.

$c_0 =$ ____

$c_1 =$ ____

$c_2 =$ ____

$c_3 =$ ____

$c_4 =$ ____

26.(1 pt) The Taylor series for $f(x) = \cos(x)$ at a = $\frac{\pi}{2}$ is $\sum_{n=0}^{\infty} c_n (x - \frac{\pi}{2})^n$.

Find the first few coefficients.

$c_0 =$ ____

$c_1 =$ ____

$c_2 =$ ____

$c_3 =$ ____

$c_4 =$ ____

27.(1 pt) The Taylor series for $f(x) = \cos(x)$ at a = $\frac{\pi}{4}$ is $\sum_{n=0}^{\infty} c_n (x - \frac{\pi}{4})^n$.

Find the first few coefficients.

$c_0 =$ ____

$c_1 =$ ____

$c_2 =$ ____

$c_3 =$ ____

$c_4 =$ ____

28.(1 pt) Find the Maclaurin series of the function $f(x) = (7x)\arctan(4x^2)$

$(f(x) = \sum_{n=0}^{\infty} c_n x^n)$

$c_3 =$ ____

$c_5 =$ ____

$c_7 =$ ____

$c_9 =$ ____

$c_{11} =$ ____

29.(1 pt) Find the Maclaurin series of the function $f(x) = (6x^2)e^{-2x}$

$(f(x) = \sum_{n=0}^{\infty} c_n x^n)$

$c_1 =$ ____

$c_2 =$ ____

$c_3 =$ ____

$c_4 =$ ____

$c_5 =$ ____

30.(1 pt) Find the Maclaurin series of the function $f(x) = (4x^2)\sin(6x)$

$(f(x) = \sum_{n=0}^{\infty} c_n x^n)$

$c_3 =$ ____

$c_4 =$ ____

$c_5 =$ ____

$c_6 =$ ____

$c_7 =$ ____

31.(1 pt) Find the Maclaurin series of the function $f(x) = 8\cos(2x^2)$

$(f(x) = \sum_{n=0}^{\infty} c_n x^n)$

$c_0 =$ ____

$c_2 =$ ____

$c_4 =$ ____

$c_6 =$ ____

$c_8 =$ ____

32.(1 pt) Match each of the Maclaurin series with right function.

— 1. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{2n+1}$

— 2. $\sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1}}{(2n+1)!}$

— 3. $\sum_{n=0}^{\infty} \frac{3^n}{n!}$

— 4. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!}$

A. $\arctan(3)$
B. $\sin(3)$
C. $\cos(3)$
D. e^3

33.(1 pt) Find $T_5(x)$: Taylor polynomial of degree 5 of the function $f(x) = \cos(x)$ at $a = 0$.

(You need to enter function.)

$T_5(x) =$ ____

Find all values of x for which this approximation is within 0.003178 of the right answer. Assume for simplicity that we limit ourselves to $|x| \leq 1$.

$|x| \leq$ ____

34.(1 pt) Let $T_6(x)$: be the Taylor polynomial of degree 6 of the function $f(x) = \cos(x)$ at $a = 0$.

Suppose you approximate $f(x)$ by $T_6(x)$, and if $|x| \leq 1$, what is the bound for your error of your estimate? (Hint: use the alternating series approximation.)

____

35.(1 pt) Let $T_k(x)$: be the Taylor polynomial of degree k of the function $f(x) = \sin(x)$ at $a = 0$.

Suppose you approximate $f(x)$ by $T_k(x)$, and if $|x| \leq 1$, how many terms do you need (that is, what is k) for you to have your error to be less than $\frac{1}{120}$? (Hint: use the alternating series approximation.)

____

36.(1 pt) Let $T_4(x)$: be the Taylor polynomial of degree 4 of the function $f(x) = \ln(1+x)$ at $a = 0$.

Suppose you approximate $f(x)$ by $T_4(x)$, find all positive values of x for which this approximation is within 0.001 of the right answer. (Hint: use the alternating series approximation.)

$0 < x \leq$ ____________________

37.(1 pt) Represent the function $5\ln(5-x)$ as a power series (Maclaurin series) $(f(x) = \sum_{n=0}^{\infty} c_n x^n)$

$c_0 =$ ____________________

$c_1 =$ ____________________

$c_2 =$ ____________________

$c_3 =$ ____________________

$c_4 =$ ____________________

Find the radius of convergence $R =$ ____________________ .

38.(1 pt) Evaluate

$$\lim_{x\to 0} \frac{\cos(x) - 1 + \frac{x^2}{2}}{15x^4}$$

Hint: Using power series.

____