

1.(1 pt) For each of the following vector fields \mathbf{F} , decide whether it is conservative or not by computing $\text{curl } \mathbf{F}$. Type in a potential function f (that is, $\nabla f = \mathbf{F}$). If it is not conservative, type N.

A. $\mathbf{F}(x,y) = (12x + 3y)\mathbf{i} + (3x + 8y)\mathbf{j}$
 $f(x,y) =$ _____

B. $\mathbf{F}(x,y) = 6y\mathbf{i} + 7x\mathbf{j}$
 $f(x,y) =$ _____

C. $\mathbf{F}(x,y,z) = 6x\mathbf{i} + 7y\mathbf{j} + \mathbf{k}$
 $f(x,y,z) =$ _____

D. $\mathbf{F}(x,y) = (6\sin y)\mathbf{i} + (6y + 6x\cos y)\mathbf{j}$
 $f(x,y) =$ _____

E. $\mathbf{F}(x,y,z) = 6x^2\mathbf{i} + 3y^2\mathbf{j} + 4z^2\mathbf{k}$
 $f(x,y,z) =$ _____

Note: Your answers should be either expressions of x , y and z (e.g. "3xy + 2yz"), or the letter "N"

2.(1 pt) If C is the curve given by $\mathbf{r}(t) = (1 + 3\sin t)\mathbf{i} + (1 + 2\sin^2 t)\mathbf{j} + (1 + 5\sin^3 t)\mathbf{k}$, $0 \leq t \leq \frac{\pi}{2}$ and \mathbf{F} is the radial vector field $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, compute the work done by \mathbf{F} on a particle moving along C .

3.(1 pt) Suppose C is any curve from $(0,0,0)$ to $(1,1,1)$ and $\mathbf{F}(x,y,z) = (3z + 4y)\mathbf{i} + (3z + 4x)\mathbf{j} + (3y + 3x)\mathbf{k}$. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

4.(1 pt) Let $\mathbf{F}(x,y) = \frac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2}$ and let C be the circle $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$, $0 \leq t \leq 2\pi$.

A. Compute $\frac{\partial Q}{\partial x}$

Note: Your answer should be an expression of x and y ; e.g. "3xy - y"

B. Compute $\frac{\partial P}{\partial y}$

Note: Your answer should be an expression of x and y ; e.g. "3xy - y"

C. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$

Note: Your answer should be a number

D. Is \mathbf{F} conservative? Type Y if yes, type N if no.

5.(1 pt) Determine whether the given set is open, connected, and simply connected. For example, if it is open, connected, but not simply connected, type "YYN" standing for "Yes, Yes, No."

A. $\{(x,y) | x > 1, y < 2\}$

B. $\{(x,y) | 2x^2 + y^2 < 1\}$

C. $\{(x,y) | x^2 - y^2 < 1\}$

D. $\{(x,y) | x^2 - y^2 > 1\}$

E. $\{(x,y) | 1 < x^2 + y^2 < 4\}$

6.(1 pt) Let C be the positively oriented circle $x^2 + y^2 = 1$. Use Green's Theorem to evaluate the line integral $\int_C 18y dx + 3x dy$.

7.(1 pt) Let C be the positively oriented square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$. Use Green's Theorem to evaluate the line integral $\int_C 3y^2 x dx + 5x^2 y dy$.

8.(1 pt) Find a parametrization of the curve $x^{2/3} + y^{2/3} = 1$ and use it to compute the area of the interior.

9.(1 pt) Let $\mathbf{F} = 6x\mathbf{i} + 3y\mathbf{j} + 3z\mathbf{k}$. Compute the divergence and the curl.

A. $\text{div } \mathbf{F} =$ _____

B. $\text{curl } \mathbf{F} =$ _____ $\mathbf{i} +$ _____ $\mathbf{j} +$ _____ \mathbf{k}

10.(1 pt) Let $\mathbf{F} = (9yz)\mathbf{i} + (9xz)\mathbf{j} + (2xy)\mathbf{k}$. Compute the following:

A. $\text{div } \mathbf{F} =$ _____

B. $\text{curl } \mathbf{F} =$ _____ $\mathbf{i} +$ _____ $\mathbf{j} +$ _____ \mathbf{k}

C. $\text{div curl } \mathbf{F} =$ _____

Note: Your answers should be expressions of x , y and/or z ; e.g. "3xy" or "z" or "5"

11.(1 pt) Let \mathbf{F} be any vector field of the form $\mathbf{F} = f(x)\mathbf{i} + g(y)\mathbf{j} + h(z)\mathbf{k}$ and let \mathbf{G} be any vector field of the form $\mathbf{F} = f(y,z)\mathbf{i} + g(x,z)\mathbf{j} + h(x,y)\mathbf{k}$. Indicate whether the following statements are true or false by placing "T" or "F" to the left of the statement.

- 1. \mathbf{G} is irrotational
- 2. \mathbf{F} is irrotational
- 3. \mathbf{G} is incompressible
- 4. \mathbf{F} is incompressible

12.(1 pt) Let $\mathbf{F} = -1y\mathbf{i} + 1x\mathbf{j}$. Use the tangential vector form of Green's Theorem to compute the circulation integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the positively oriented circle $x^2 + y^2 = 4$.

13.(1 pt) Let $\mathbf{F} = 5x\mathbf{i} + 3y\mathbf{j}$ and let \mathbf{n} be the outward unit normal vector to the positively oriented circle $x^2 + y^2 = 16$. Compute the flux integral $\int_C \mathbf{F} \cdot \mathbf{n} ds$.

14.(1 pt) A rock with a mass of 15 kilograms is put aboard an airplane in New York City and flown to Boston. How much work does the gravitational field of the earth do on the rock?
 _____ Newton-meters

15.(1 pt) Suppose $\mathbf{F} = \mathbf{F}(x,y,z)$ is a gradient field with $\mathbf{F} = \nabla f$, S is a level surface of f , and C is a curve on S . What is the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$?

16.(1 pt)

A vector field gives a geographical description of the flow of money in a society. In the neighborhood of a political convention, the divergence of this vector field is:

- A. positive
- B. negative
- C. zero