

1.(1 pt) Evaluate  $\iint_S \sqrt{1+x^2+y^2} dS$  where  $S$  is the helicoid:  $\mathbf{r}(u,v) = u\cos(v)\mathbf{i} + u\sin(v)\mathbf{j} + v\mathbf{k}$ , with  $0 \leq u \leq 3, 0 \leq v \leq 1\pi$

2.(1 pt) Find the surface area of the part of the sphere  $x^2 + y^2 + z^2 = 25$  that lies above the cone  $z = \sqrt{x^2 + y^2}$

3.(1 pt) A fluid has density 3 and velocity field  $\mathbf{v} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ . Find the rate of flow outward through the sphere  $x^2 + y^2 + z^2 = 9$

4.(1 pt) Let  $S$  be the part of the plane  $2x + 1y + z = 4$  which lies in the first octant, oriented upward. Find the flux of the vector field  $\mathbf{F} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$  across the surface  $S$ .

5.(1 pt) Use Gauss's law to find the charge enclosed by the cube with vertices  $(\pm 1, \pm 1, \pm 1)$  if the electric field is  $\mathbf{E}(x,y,z) = 2x\mathbf{i} + 5y\mathbf{j} + z\mathbf{k}$ .

6.(1 pt) The temperature  $u$  in a star of conductivity 3 is inversely proportional to the distance from the center:  $u = \frac{\epsilon_0}{\sqrt{x^2+y^2+z^2}}$ . If the star is a sphere of radius 1, find the rate of heat flow outward across the surface of the star.

7.(1 pt) Use Stoke's theorem to evaluate  $\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$  where  $\mathbf{F}(x,y,z) = -12yz\mathbf{i} + 12xz\mathbf{j} + 12(x^2+y^2)z\mathbf{k}$  and  $S$  is the part of the paraboloid  $z = x^2 + y^2$  that lies inside the cylinder  $x^2 + y^2 = 1$ , oriented upward.

8.(1 pt) Use Stoke's Theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + 1(x^2 + y^2)\mathbf{k}$  and  $C$  is the boundary of the

part of the paraboloid where  $z = 1 - x^2 - y^2$  which lies above the  $xy$ -plane and  $C$  is oriented counterclockwise when viewed from above.

9.(1 pt) Use the divergence theorem to find the outward flux of the vector field  $\mathbf{F}(x,y,z) = 3x^2\mathbf{i} + 5y^2\mathbf{j} + 1z^2\mathbf{k}$  across the boundary of the rectangular prism:  $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 1$ .

10.(1 pt) If a parametric surface given by  $\mathbf{r}_1(u,v) = f(u,v)\mathbf{i} + g(u,v)\mathbf{j} + h(u,v)\mathbf{k}$  and  $-4 \leq u \leq 4, -2 \leq v \leq 2$ , has surface area equal to 3, what is the surface area of the parametric surface given by  $\mathbf{r}_2(u,v) = 2\mathbf{r}_1(u,v)$  with  $-4 \leq u \leq 4, -2 \leq v \leq 2$ ?

11.(1 pt) Suppose  $\mathbf{F}$  is a radial force field,  $S_1$  is a sphere of radius 7 centered at the origin, and the flux integral  $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = 4$ .

Let  $S_2$  be a sphere of radius 49 centered at the origin, and consider the flux integral  $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$ .

(A) If the magnitude of  $\mathbf{F}$  is inversely proportional to the square of the distance from the origin, what is the value of  $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$ ?

(B) If the magnitude of  $\mathbf{F}$  is inversely proportional to the cube of the distance from the origin, what is the value of  $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$ ?

12.(1 pt)

In springtime, the average value over time of the divergence of the vector field which represents air flow is:

- A. zero
- B. positive
- C. negative