

1.(1 pt) Suppose $f(x,y) = x^2 + y^2 - 8x - 8y + 1$
 (A) How many critical points does f have in \mathbf{R}^2 ?

(B) If there is a local minimum, what is the value of the discriminant D at that point? If there is none, type N.

(C) If there is a local maximum, what is the value of the discriminant D at that point? If there is none, type N.

(D) If there is a saddle point, what is the value of the discriminant D at that point? If there is none, type N.

(E) What is the maximum value of f on \mathbf{R}^2 ? If there is none, type N.

(F) What is the minimum value of f on \mathbf{R}^2 ? If there is none, type N.

2.(1 pt) Suppose $f(x,y) = xy - ax - by$.
 (A) How many local minimum points does f have in \mathbf{R}^2 ?
 (The answer is an integer).

(B) How many local maximum points does f have in \mathbf{R}^2 ?

(C) How many saddle points does f have in \mathbf{R}^2 ?

3.(1 pt) Consider the function $f(x,y) = x \sin(y)$.
 In the following questions, enter an integer value or type INF for infinity.

(A) How many local minima does f have in \mathbf{R}^2 ?

(B) How many local maxima does f have in \mathbf{R}^2 ?

(C) How many saddle points does f have in \mathbf{R}^2 ?

4.(1 pt) Suppose $f(x,y) = xy(1 - 5x - 3y)$.
 $f(x,y)$ has 4 critical points. List them in increasing lexicographic order. By that we mean that (x, y) comes before (z, w) if $x < z$ or if $x = z$ and $y < w$. Also, describe the type of critical point by typing MA if it is a local maximum, MI if it is a local minimum, and S if it is a saddle point.

First point (_____ , _____) of type _____
 Second point (_____ , _____) of type _____
 Third point (_____ , _____) of type _____
 Fourth point (_____ , _____) of type _____

5.(1 pt) Each of the following functions has at most one critical point. Graph a few level curves and a few gradients and, on this basis alone, decide whether the critical point is a local maximum (MA), a local minimum (MI), or a saddle point (S). Enter

the appropriate abbreviation for each question, or N if there is no critical point.

(A) $f(x,y) = e^{-4x^2-3y^2}$

Type of critical point: _____

(B) $f(x,y) = e^{4x^2-3y^2}$

Type of critical point: _____

(C) $f(x,y) = 4x^2 + 3y^2 + 4$

Type of critical point: _____

(D) $f(x,y) = 4x + 3y + 4$

Type of critical point: _____

6.(1 pt) You are to manufacture a rectangular box with 3 dimensions x, y and z , and volume $v = 4913$. Find the dimensions which minimize the surface area of this box.

$x =$ _____
 $y =$ _____
 $z =$ _____

7.(1 pt) Find the coordinates of the point (x, y, z) on the plane $z = 2x + 3y + 1$ which is closest to the origin.

$x =$ _____
 $y =$ _____
 $z =$ _____

8.(1 pt) Find the maximum and minimum values of $f(x,y) = 8x^2 + 9y^2$ on the disk $D: x^2 + y^2 \leq 1$.

maximum value: _____

minimum value: _____

9.(1 pt) Find the maximum and minimum values of $f(x,y) = 7x + y$ on the ellipse $x^2 + 49y^2 = 1$

maximum value: _____

minimum value: _____

10.(1 pt) For each of the following functions, find the maximum and minimum values of the function on the circular disk: $x^2 + y^2 \leq 1$. Do this by looking at the level curves and gradients.

(A) $f(x,y) = x + y + 3$:

maximum value = _____

minimum value = _____

(B) $f(x,y) = 3x^2 + 4y^2$:

maximum value = _____

minimum value = _____

(C) $f(x,y) = 3x^2 - 4y^2$:

maximum value = _____

minimum value = _____

11.(1 pt) For each of the following functions, find the maximum and minimum values of the function on the rectangular region: $-3 \leq x \leq 3, -4 \leq y \leq 4$.

Do this by looking at level curves and gradients.

(A) $f(x,y) = x + y + 3$:

maximum value = _____

minimum value = _____

(B) $f(x,y) = 3x^2 + 4y^2$:

maximum value = _____

minimum value = _____

$$(C) = f(x, y) = (4)^2x^2 - (3)^2y^2:$$

maximum value = _____

minimum value = _____

12.(1 pt) Find the maximum and minimum values of $f(x, y, z) = 1x + 3y + 1z$ on the sphere $x^2 + y^2 + z^2 = 1$.

maximum value = _____

minimum value = _____

13.(1 pt) Find the maximum and minimum values of $f(x, y) = xy$ on the ellipse $3x^2 + y^2 = 5$.

maximum value = _____

minimum value = _____

14.(1 pt) You are hiking the Inca Trail on the way to Machu Piccho. When you arrive at the highest point on the trail, which of the following are possibilities? In alphabetical order without punctuation or spacing, list the letters which indicate possibilities.

(A) The path passes through the center of a set of concentric contour lines.

(B) The path is tangent to a contour line.

(C) The path follows a contour line.

(D) The path crosses a contour line.

possibilities: _____