Math 217 - Practice 1

Notes to the reader: This practice exam is meant to prepare you for the first midterm exam by showing you a variety of problems that will cover many of the concepts to learn for the exam. However, that does not mean that these are the only problems to study for the test - in general, these problems may or may not be similar to what appears on the exam. You should also not use these problems or this practice as a gauge of the actual exam's difficulty. I strongly encourage you to try these problems *without* using a calculator, your notes, or any other references.

The actual exam will consist of 10 multiple choice questions worth 7% each, as well as 2 hand graded questions worth 15% each. You should make sure that you can write clear and concise solutions to each of these problems!

(1) Write an equation of the form $y'' + Ay' + By = 0$, with A and B constants, such that $y(x) = \cos(3x) + \sin(3x)$ is a solution.

(2) If the acceleration of an object is $a(t) = 4(t+3)^2$, and its initial velocity is -1 and its initial position is 1, where is the object at time t?

(3) Draw a careful slope field and identify some sample trajectories for $\frac{dy}{dx} = x^2 - y$.

(4) Draw a careful slope field and identify some sample trajectories for $y' = (y-1)(y+3)^2$. Identify possible values for $\lim_{t\to\infty} y(t)$ based on the initial condition.

(5) Determine whether an equation is guaranteed to have a unique solution through a particular point: (a) $dy/dx = x \ln y$ through $(1, 0)$; (b) $e^y y' + y = \tan x$ through $(0, 0)$; (c) $y' = 3y^{2/3}$ through the point (a, b).

(6) Identify the type of equation (linear, exact, homogeneous, Bernoulli, etc.) and solve it. Some can be solved with multiple techniques; in that case, use each technique that is applicable!

(a) $y' = 2x \sec y, y(0) = 0$; compute $y(1/\sqrt{2})$

(b) $y' = xy^2 + x, y(0) = 0$; compute $y(1)$

(c) $dy/dx = 4x^3 y - y, y(1) = -3$

(d) $xy' + 2y = 6x^2 \sqrt{y}$

(e) $e^y + y \cos x + (e^y + \sin x)y' = 0$

(f) $dy/dx = (2xy + 2x)/(x^2 + 1)$

(g) $dy/dx = (\sqrt{y} - y)/\tan x$

(h) $xy' = 2y + x^3 \cos x$

(7) A 100 gallon tank initially contains 50 gallons of pure water. Brine with a salt concentration of 2 pounds/gallon is pumped in at a rate of 5 gallons per minute. Well-stirred mixture drains from the tank at a rate of 4 gallons per minute. After 50 minutes the tank is full; how much salt is contained in the tank at this time?

(8) A population is modeled by $P' = 0.01P(500 - P)$ with $P(0) = 100$. Interpret this model (that is, explain what the numbers mean)

This document contains the answers to the computational problems in the practice document for Math 217 Exam 1. In case of typos, please let me know.

(1) $y'' + 9y = 0$ is a possible answer, as can be verified by direct computation of y''.

(2) The position satisfies the initial value problem

$$s''(t) = 4(t+3)^2, \quad s(0) = 1, s'(0) = -1.$$

After integrating twice, we find that $s(t) = \frac{1}{3}t^4 + 4t^3 + 18t^2 - t + 1$. If you left your answer in factored form, that is also fine.

(3) Visualize the slope field with a calculator, e.g. at https://www.geogebra.org/m/W7dAdgqc.

(4) Same as (3). The limiting values are $y = 1, y = -3$, as well as $\pm\infty$.

(5) (a) does not have a solution, because $\ln(0)$ is undefined. (b) has a unique solution because it can be written as $y' = ye^{-y} + e^{-y}\tan x$, which is continuously differentiable in y. (c) depends on the initial condition, and has a unique solution when $b \neq 0$. This is a result of the fact that $\partial_y 3y^{2/3} = 2y^{-1/3}$ is continuous unless $y = 0$.

(6) (a) $\pi/6$
  (b) $\tan(1/2)$
  (c) $y(x) = -3e^{x^4 - x}$
  (d) $y(x) = 2Cx + C^2x^{-2} + x^4$. If you have an implicit form of the answer, that is fine (just check equivalence!).
  (e) The original equation should have read $e^y + y\cos x + (xe^y + \sin x)y' = 0$, in which case it is exact. Note the missing x in the third term. The potential function is $xe^y + y\sin x = C$.
  (f) $y(x) = C(x^2+1) - 1$
  (g) $y(x) = 1 + 2e^{C/2}/\sqrt{\sin x} + e^C/\sin x$. Again, this is probably better left as an implicit solution.
  (h) $y(x) = Cx^2 + x^2 \sin x$

(7) The differential equation is

$$Q'(t) = 10 - \frac{4Q}{50+t}$$

with the initial condition $Q(0) = 0$. The solution is $Q(t) = 2(50+t)^4 - 2 \cdot 50^5/(50+t)^4$, and so the quantity is

$$Q(50) = 200 - \frac{2 \cdot 50^4}{100^4} = 200 - \frac{50}{8} = 193.75$$

(8) This is a logistic equation with a starting population of 100. The carrying capacity in this case is 500, and the population will increase over time towards this (stable) equilibrium.