Math 217 - Practice 2

Notes to the reader: This practice exam is meant to prepare you for the second midterm exam by showing you a variety of problems that will cover many of the concepts to learn for the exam. However, that does not mean that these are the only problems to study for the test - in general, these problems may or may not be similar to what appears on the exam. You should also not use these problems or this practice as a gauge of the actual exam's difficulty. I strongly encourage you to try these problems *without* using a calculator, your notes, or any other references.

The actual exam will consist of 9 multipe choice questions worth 6% each, as well as 3 hand graded questions making up the remaining 46% of the exam. You should make sure that you can write clear and concise solutions to each of these problems!

(1) Compute the Wronskian of $\sin x, \cos x$, and e^x. Are the functions linearly independent?

(2) Determine if $x, \cos(\ln x), \sin(\ln x)$ are linearly independent on $(0, \infty)$.

(3) Write a non-trivial linear combination of $x, 3x + x^2$, and $x^2 - 14x$ which is zero on all of $\mathbb{R}$.

(4) Solve the initial value problem $9y'' + 6y' + 4y = 0; y(0) = 3, y'(0) = 4$.

(5) Solve the initial value problem $3y^{(3)} + 2y'' = 0; y(0) = -1, y'(0) = 0, y''(0) = 1$.

(6) Solve the initial value problem $y'' - 4y' + 3y = 0; y(0) = 7, y'(0) = 11$.

(7) Write a constant coefficient linear homogeneous equation whose general solution is $A \cos 2x + B \sin 2x + Ce^x + D + Ex$.

(8) Determine the period and frequency of the motion of a 4-kg mass on the end of a spring with spring constant 16 N/m. If the mass starts at position +2 m with velocity −4 m/s, write the position in the form

$$x(t) = A \cos(\omega_0 t - \delta).$$

(9) Suppose that the parameters in a free mass-spring-dashpot system are $m = 10$, $c = 9$, $k = 2$. The mass is set in motion with $x(0) = 0$ and $x'(0) = 5$. Find the position function $x(t)$; what is the farthest position the mass reaches?

(10) Find a particular solution to $y^{(3)} - y = e^x + 7$.

(11) Find a particular solution to $y^{(3)} + y' = 2 - \sin x$.

(12) Find a particular solution to $y^{(4)} - 2y'' + y = xe^x$.

(13) Find the steady periodic solution and the transient solution for $x''+4x'+5x = 10 \cos 3t; x(0) = x'(0) = 0$.

(14) In the previous problem, set the damping to zero. What would the resonant frequency be? Solve the same IVP with the force function $F(t) = \cos \omega t$, where ω is the resonant frequency.

Answers.

(1) The Wronskian is $-2e^x$, and yes.

(2) Yes, because the Wronskian is $-2/x^2$.

(3) $1(x^2 - 14x) + (-1)(x^2 + 3x) + (-11)x = 0$

(4) $y = e^{-x/3}\left(5\sqrt{3}\sin\frac{x}{\sqrt{3}} + 3\cos\frac{x}{\sqrt{3}}\right)$

(5) $y = -\frac{13}{4} + \frac{3}{2}x + \frac{9}{4}e^{-2x/3}$

(6) $y = 2e^{3x} + 5e^{x}$

(7) The characteristic polynomial is $(r-2i)(r+2i)(r-1)r^2$, so the equation is $y^{(5)} - y^{(4)} + 4y^{(3)} - 4y'' = 0$.

(8) The differential equation is $4x'' + 16x = 0, x(0) = 2, x'(0) = -4$. The position is $2\cos(2t) - 2\sin(2t) = 2\sqrt{2}\cos(2t - \pi/4)$, the circular frequency is 2 and the period is π; the amplitude is $2\sqrt{2}$.

(9) The position is $50e^{-2t/5} - 50e^{-t/2}$ and the maximum displacement is approximately 4.1.

(10) $\frac{1}{3}xe^x + 7$

(11) $2x + \frac{1}{2}x\sin x$

(12) $\frac{1}{24}x^3e^x - \frac{1}{8}x^2e^x$

(13) The steady period solution is $\frac{\sqrt{10}}{4}\cos(3t - \alpha)$ with $\alpha = \pi - \arctan(3)$. The transient solution is $\frac{5}{4}\sqrt{2}e^{-2t}\cos(t - \beta)$ with $\beta = 2\pi - \arctan(7)$.

(14) The resonant frequency for $x'' + 5x$ is $\sqrt{5}$; the solution is $\frac{1}{2\sqrt{5}}t\sin(\sqrt{5}t)$ which grows in amplitude over time.