

Homework 3

Math 217

Due: 26 September 2018 by 11:00 PM

Instructions: Write your solutions to the following problems and submit them on Crowdmark by the deadline. You are encouraged to work in groups or consult with each other on the problems, but the work submitted must be your own and must be written up by you.

(1) (Variant of 3.1 #18) Show that $y = x^3$ is a solution of $yy'' = 6x^4$, but that $y = cx^3$ is not a solution when $c \neq \pm 1$. Why does this not contradict our results about superposition?

(2) Solve the initial value problem

$$y'' + 2y' - 15y = 0$$

with the initial conditions

$$y(0) = 2, \quad y'(0) = -1.$$

(3) (Variant of 3.2 #27) Show that the functions $f_1(x) = 1$, $f_2(x) = x$, and $f_3(x) = x^2$ are linearly independent in two ways: first by computing the Wronskian, and second by using the definition of linear independence directly. As a hint, once you have the equation $c_1 + c_2x + c_3x^2$, differentiate up to two times and plug in carefully chosen values of x to show that $c_1 = c_2 = c_3 = 0$.

(4) A frequent technique used to study second order equations is *reduction of order*. If we can make an educated guess of one solution, then we can frequently use it to build a second solution. Consider the equation

$$x^2y'' - 3xy' + 4y = 0.$$

(a) Find a solution to this equation of the form $y_1(x) = x^r$.

(b) Assume that the second equation is of the form $y_2(x) = v(x)y_1(x)$ for an unknown function v . Using your answer from part (a), substitute this in to the second order equation to show that $xv'' + v' = 0$.

(c) Find the general solution to the equation $x^2y'' - 3xy' + 4y = 0$.