Homework 9

Math 217

Due: 28 November 2018 by 11:59 PM

Instructions: Write your solutions to the following problems and submit them on Crowdmark by the deadline. You are encouraged to work in groups or consult with each other on the problems, but the work submitted must be your own and must be written up by you.

(1) Find the solution to the system

$$\begin{cases} x' = -y \\ y' = 13x + 4y \end{cases}$$

with initial conditions $x(0) = 0, y(0) = 3$.

(2) Rewrite the third-order constant coefficient equation

$$y^{(3)} + ay'' + by' + cy = f(t)$$

as a 3×3 matrix system $\vec{v}' = A\vec{v} + \vec{b}$.

(3) Three masses are connected by four springs in the following setup:

W – S – M – S – M – S – M – S – W

where W is a wall, S is a spring, and M is a mass (see 4.2 problem 47 for a better diagram!). If the masses are all 1 and the spring constants are all 1, show that the natural frequencies of oscillation of the system are $\sqrt{2} \approx 1.41$, $\sqrt{2-\sqrt{2}} \approx 0.77$, and $\sqrt{2+\sqrt{2}} \approx 1.85$. Start by setting up a system of three equations for the positions of each mass, making sure to justify the form of the equation that you get.

(4) If a particle with mass m is given a charge q, then the force it experiences from a magnetic field $\vec{B}$ is given by $\vec{F} = q\vec{v} \times \vec{B}$, where $\times$ is the vector cross product. Suppose that the magnetic field is of the form $\vec{B} = B\vec{k}$ (purely in the z-direction).

(a) Recalling that force is mass times acceleration, write a system of two (second order) differential equations for the position $(x(t), y(t))$.

(b) If we have initial conditions $x(0) = r_0, y(0) = 0, x'(0) = 0, y'(0) = -qBr_o/m$, show that the particle moves in a circle with radius r_0.

Some suggestions, updated 11/26: For problem (4), the motion in the z direction is purely linear (and you should at some point have an equation $z'' = 0$). Feel free to either ignore this, or notice that while the motion in the xy-plane is indeed circular, the overall motion is helical. Add conditions $z(0) = z'(0) = 0$ if you desire.

For problem (3), you can solve for x via elimination. Take two derivatives at a time; you should have an intermediate step where you get an equation similar to $x^{(4)} + 4x'' + 3x = z$. Then differentiate twice again and make the appropriate substitutions. You may wish to use (but make sure you explain why these are true!) the facts that $y = x'' + 2x$ and $y'' = x^{(4)} + 2x''$. You'll end up with a sixth order equation, and feel free to use a calculator/computer to locate the roots of the characteristic polynomial.