Wednesday 10/10 : Laplace transform.

Given f, its Laplace transform is

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty \underbrace{e^{-st}}_{\text{kernel of the operator.}} f(t)\,dt$$

Ex. $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$ is valid for $s > a$.

Key property : $\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$.

So Laplace turns d/dx into multiplication by s, which is an algebraic operation.

Ex. $y' - y = e^{2t}$, $y(0) = 0$

$$\downarrow$$

$$sY(s) - Y(s) = \frac{1}{s-2}$$

$$\downarrow$$

$$Y(s) = \frac{1}{(s-2)(s-1)} = \frac{-1}{s-1} + \frac{1}{s-2}$$

$$\therefore\ y(t) = -\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= -e^{t} + e^{2t}.$$

Algorithm: 1) Use $\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$ to find Laplace transform of a differential equation.

2) Rearrange $Y(s)$ into a recognizable form.

3) Invert the transform.