Friday 10/12: Transforming IVPs.

Key fact: $\mathcal{L}\{y'\} = sY(s) - y(0)$

$$\downarrow$$

$$\mathcal{L}\{y''\} = s^2 Y(s) - sy(0) - y'(0)$$

and so on. So we turn ODEs into purely algebraic equations.

Ex. $y'' - 4y' = e^t$, $y(0) = y'(0) = 0$

1) Translate to Laplace side

$$s^2 Y(s) - 4sY(s) = \frac{1}{s-1}$$

$$\longrightarrow \quad Y(s) = \frac{1}{(s^2-4s)(s-1)}$$

2) Simplify into known form (typically by partial fractions)

$$Y(s) = \frac{1}{s(s-1)(s-4)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-4}$$

$$= -\frac{1}{3}\,\frac{1}{s-1} + \frac{1}{4}\,\frac{1}{s} + \frac{1}{12}\,\frac{1}{s-4}$$

3) Invert the transform:

$$y(t) = \underbrace{-\frac{1}{3}e^{t}}_{\text{particular}} + \underbrace{\frac{1}{4} + \frac{1}{12}e^{4t}}_{\text{complementary.}}$$

Note: You will be given a table of transforms on the exams as needed - no memorizing!