Monday October 1 — Undetermined Coefficients.

Our goal is to solve $Ly = f(x)$ where L is a linear differential operator. We build a particular solution from $f, f', f'', \ldots$ by collecting all linearly independent terms.

Ex. $y'' - 2y' + 5y = x^2$

  then use $x^2, 2x, 2 \longrightarrow Ax^2 + Bx + C$

Ex. $y'' - 2y' + 5y = e^x + \cos x$

  then use $e^x, \cos x, \sin x \longrightarrow Ae^x + B\cos x + C\sin x$

Ex. $y'' - y = x \sin x$

  then use that $\underset{=}{x \sin x} \xrightarrow{d/dx} \underset{=}{x \cos x} + \underset{=}{\sin x}$

  $\xrightarrow{d/dx} -x\sin x + 2\cos x$

  $\xrightarrow{d/dx} -(x\cos x + \sin x) - 2\sin x$

  etc. All derivatives are built from the blocks highlighted.

  $\longrightarrow y_p = A\sin x + B\cos x + Cx\sin x + Dx\cos x.$

Duplication: If a term solves the homogeneous equation, multiply by x. Compare to reduction of order & repeated roots.

Ex. $y'' - 9y = e^{3x}$. Complementary solution

  is $y_c = C_1 e^{3x} + C_2 e^{-3x} \longrightarrow y_p = Axe^{3x}.$