Monday 10/22 - Discontinuous Sources.

Recall the step function $u(t-a) = \begin{cases} 1 & t \geq a \\ 0 & t < a \end{cases}$

represents power being turned on at time a. Then

$$\mathcal{L}\{u(t-a)\, f(t-a)\} = e^{-as} F(s)$$

$$\mathcal{L}^{-1}\{e^{-as} F(s)\} = u(t-a)\, f(t-a).$$

Ex. if $f(t) = \begin{cases} t^2 & t \geq 2 \\ 0 & t < 2 \end{cases}$ then we can write

$$f(t) = \left[(t-2)^2 + 4(t-2) + 4\right] u(t-2)$$

$$\therefore \mathcal{L}\{f(t)\} = \left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right] e^{-2s}.$$

Ex. $x'' + 4x = (1 - u(t-2\pi))\cos 2t$: forcing turns off at 2π.

Then $$X(s) = \frac{1}{s^2+4}\left[\frac{s}{s^2+4} - \frac{s e^{-2\pi s}}{s^2+4}\right]$$

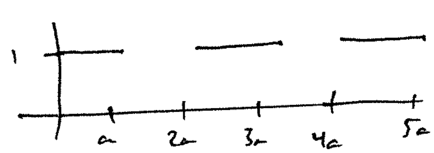
$$x(t) = \frac{1}{4} t \sin 2t - u(t-2\pi) \cdot \frac{1}{4}(t-2\pi)\sin(2(t-2\pi))$$

growing wave while forced, steady afterwards.

$$= \begin{cases} \frac{1}{4} t \sin 2t & 0 \leq t \leq 2\pi \\ \frac{\pi}{2} \sin 2t & t > 2\pi \end{cases}$$

Can also apply to periodic sources.

Ex. Square wave



$$= \sum_{n=0}^{\infty} (-1)^n u(t-na).$$

Then $$\mathcal{L}\{\text{square wave}\} = \sum_{n=0}^{\infty} (-1)^n \mathcal{L}\{u(t-na)\} = \frac{1}{s(1+e^{-as})}.$$