Wednesday 10/24. Impulse functions.

We can imagine a momentary impulse (e.g. inelastic collision) as the limit of shorter & shorter pulses.

Take $d_{a,\epsilon}(t) = \begin{cases} 1/\epsilon & t \in [a, a+\epsilon] \\ 0 & \text{else.} \end{cases}$

Then $\int_0^\infty d_{a,\epsilon}(t)\,dt = 1$ for all $\epsilon > 0$.

But $\lim_{\epsilon \to 0} d_{a,\epsilon}(t) = \begin{cases} \infty & t = a \\ 0 & \text{else} \end{cases}$ doesn't make sense!

Think about operators instead. Can write $\int_0^\infty g(t)\, d_{a,\epsilon}(t)\,dt$

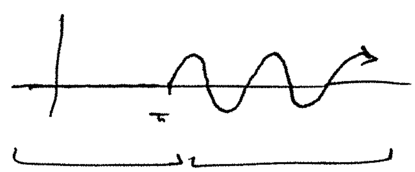
as $\frac{1}{\epsilon}\int_a^{a+\epsilon} g(t)\,dt \longrightarrow g(a)$

Dirac Delta is defined by $\int_0^\infty \delta_a(t)\, f(t)\,dt = f(a)$

or $\int_0^\infty \delta(t-a)\, f(t)\,dt = f(a) = \underline{\underline{f * \delta}}$

Then $\mathcal{L}\{\delta_a\} = e^{-as} \longrightarrow$ gives step functions.

Ex. $x'' + 9x = \delta(t-\pi) \longrightarrow X(s) = \dfrac{e^{-\pi s}}{s^2+9} \longrightarrow x(t) = \frac{1}{3}\sin[3(t-\pi)]\,u(t-\pi)$



zero steady oscillation.

Key result is Duhamel's Principle:

$$x(t) = \begin{pmatrix} \text{response to} \\ \delta \text{ impulse} \end{pmatrix} * \begin{pmatrix} \text{driving} \\ \text{force} \end{pmatrix}$$

$\hookrightarrow$ MEASURABLE without knowing the underlying system!!