Monday 10/29 - Series Solutions.

We're going to look for series solutions to ODE's.

Ex. $y' - 3y = 0$. Use $y = \sum_{n=0}^{\infty} c_n x^n$.

If we differentiate term-by-term, get

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

after shifting index. Then

$$y' - 3y = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n - 3\left(\sum_{n=0}^{\infty} c_n x^n\right)$$

$$= \sum_{n=0}^{\infty} \left[(n+1) c_{n+1} - 3c_n\right] x^n = 0 = \sum_{n=0}^{\infty} 0 x^n$$

By the identity principle, $(n+1) c_{n+1} - 3c_n = 0$ for all n.

This gives a recurrence relation

$$c_{n+1} = \frac{3c_n}{n+1}.$$

If we solve it, $c_n = \frac{3^n c_0}{n!}$

$$\therefore \quad y = \sum_{n=0}^{\infty} \frac{3^n c_0}{n!} x^n = c_0 e^{3x}.$$

This is a typical format. We get a relationship that determines the coefficients. Practically, only a few terms are necessary for (reasonably) high-precision computations - great for approximations!