10/31 Wednesday - Power Series, continued.

Power series are only useful if the converge. We can tell from the ratio test.

Thm. The series $\sum_{n=0}^{\infty} c_n x^n$ converges for all x with

$$|x| < \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| \quad \text{and diverge for} \quad |x| > \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$$

provided the limit exists or is $+\infty$.

Ex. $y' - 3y = 0 \longrightarrow c_n = \frac{3^n}{n!} \longrightarrow \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = \underline{\underline{\infty}}$

Singularity @ $x=1$! Ex. $(x-1)y' + 2y = 0 \longrightarrow c_n = n+1 \longrightarrow \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = \underline{\underline{1}}$.

This limit is the radius of convergence.

We can also solve second order equations.

Ex. $y'' + y = 0$. We got the recurrence

$$c_{n+2} = \frac{-c_n}{(n+1)(n+2)}$$

Even terms: $c_{2n} = \frac{(-1)^n c_0}{(2n)!}$ Odd terms: $c_{2n+1} = \frac{(-1)^n c_1}{(2n+1)!}$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = \cos x \qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = \sin x.$$

So we get $y = \underbrace{c_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}}_{\cos x} + \underbrace{c_1 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}}_{\sin x}$

Two linearly independent solutions!