Wednesday October 3. Forced Oscillation & Resonance.

$$mx'' + cx' + kx = F(t):$$ Now we can handle F!

<u>Ex.</u> No damping, $F = F_0 \cos \omega t$.

$$\hookrightarrow \quad x'' + \frac{k}{m} x = F_0 \cos \omega t$$

$$x'' + \omega_0^2 x = F_0 \cos \omega t$$

ω_0 = natural response frequency of system

Solve with undetermined coefficients ...

$$x(t) = \underbrace{C_1 \cos \omega_0 t + C_2 \sin \omega_0 t}_{\text{Natural response}} + \underbrace{\frac{F_0/k}{1-\left(\frac{\omega}{\omega_0}\right)^2} \cos \omega t}_{\text{Amplified!}}$$

If $\omega \approx \omega_0$, the response is <u>large</u> and the input signal is greatly amplified. Damping decreases this:

<u>Ex.</u> $x'' + 2px' + \omega_0^2 x = \frac{F_0}{m} \cos \omega t$.

Underdamped case, e.g. two complex roots: Solution is of the form

$$x = \underbrace{C_1 \left(e^{-pt} \cos \ldots t\right) + C_2\left(e^{-pt} \sin \ldots t\right)}_{\substack{\text{transient - amplitude} \to 0 \\ \text{due to exponentials.}}} + \underbrace{C_3 \cos \omega t + C_4 \sin \omega t}_{\substack{\text{Steady periodic} \\ \equiv \text{Long-term} \\ \text{behavior.}}}$$

The closer ω is to the ~~natural frequency~~ practical resonant, the stronger the response. Applications: Radio tuners, driving on a washboard road, structural failures, etc.!