

Wednesday 11/14 - Systems of Equations.

A system of differential equations is a set of differential equations relating two or more unknown functions.

Ex.
$$\begin{aligned}x' &= y + 1 \\y' &= x + y\end{aligned}$$
 first order linear
2x2 system

Ex
$$\begin{aligned}z' &= x' + xy \\y' &= xyz \\x' &= x + y + z\end{aligned}$$
 first order nonlinear
3x3 system.

We can frequently translate $n \times n$ systems into n^{th} order equations via elimination.

Ex.
$$\begin{aligned}x' &= y \\y' &= -x\end{aligned}$$
 $\rightarrow x'' = (y)' = -x$
 $\rightarrow x'' + x = 0$
 $\rightarrow x = A \cos t + B \sin t$
 $y = -A \sin t + B \cos t.$

Interpretation: $(x(t), y(t))$ is a trajectory that comes from the level set of $z = x^2 + y^2$ ($\frac{dz}{dt} = 0$).

Can visualize with a phase portrait.