Friday 11/16 - <u>Elimination methods</u>

Can generally solve by elimination (compare to solving algebraic systems!). Substitute to eliminate a variable.

<u>Ex.</u> $x' = y + x$

$y' = 3x - y$

$$\begin{aligned} x'' &= y' + x' \\ &= (3x - y) + x' \\ &= 3x - (\underbrace{x' + x}_{\text{from } 1^{st} \text{ equation}}) + x' \\ &= 2x. \end{aligned}$$

Now we can get $x'' - 2x = 0$.

Main goal: Translate from n^{th} order to 1^{st} order system.

Matrix notation is useful: $x' = c_1 x + c_2 y + f(t)$

$y' = c_3 x + c_4 y + g(t)$

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} f \\ g \end{bmatrix}.$$

We can turn $y'' + ay' + by = f$ into a system

$u' = v$, $v' = u'' = -by - ay' + f = -bu - av + f$.

$$\rightsquigarrow \begin{bmatrix} u \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ f \end{bmatrix}.$$

Why is this important? Because we can solve

$\vec{x}' = A\vec{x}$ via $\vec{x} = e^{At}\vec{x}(0)$

with the <u>matrix exponential.</u>