Monday 4/26 - Matrix Systems

We've solve systems via elimination, but now we want to view these internally as first order equations, without translating to high-order equations. So we need some algebra.

- Sums & differences are done component-wise.
- Products are more complicated: Given a matrix A and a matrix B, the (i,j) component (row i, column j) of AB is $\begin{pmatrix} \text{row } i \\ \text{of } A \end{pmatrix} \cdot \begin{pmatrix} \text{column } j \\ \text{of } B \end{pmatrix}$.
- The inverse of A is a matrix B so $AB = BA = I_n$, the $n \times n$ identity matrix, which has an algebraic role similar to 1.

  An $n \times n$ matrix A is invertible $\iff \det A \neq 0$

  $\iff A\vec{v} = 0$ only has $\vec{v} = 0$ as a solution.

To solve $\vec{x}\,' = A\vec{x}$, look for a solution $\vec{x} = \vec{v}e^{\lambda t}$. Get $\lambda \vec{v} e^{\lambda t} = A\vec{v}e^{\lambda t}$,

so we want $\boxed{A\vec{v} = \lambda \vec{v}}$.

A non-zero $\vec{v}$ with this property is called an eigenvector. λ is its eigenvalue. Next, we'll try to find these pairs.