Wednesday 11/28 - Eigenvalues & Eigenvectors

Recall we want to solve $\vec{x}' = A\vec{x}$, so we're looking for a solution $\vec{x} = \vec{v}e^{\lambda t}$ where $\vec{v}, \lambda$ are an eigenvector/eigenvalue pair. If $A\vec{v} = \lambda\vec{v}$, $(A - \lambda I)\vec{v} = 0$ and so we want $A - \lambda I$ non-invertible. So

$$\boxed{\det(A - \lambda I) = 0}$$

Characteristic Polynomial.

This is a degree $n$ polynomial!

Ex. $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \longrightarrow (2-\lambda)(1-\lambda) - 3\cdot 2 = 0$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = -1, \ 4.$$

Note:
$\Sigma$ eigenvalues = trace,
$\Pi$ eigenvalues = determinant

Eigenvectors: Want $(A - 4I)\vec{v} = 0$

$$\rightsquigarrow \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \vec{0}$$

$-2a + 3b = 0$, $b = \frac{2}{3}a$. So $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ works.

Also need $(A - (-1)I)\vec{v} = 0$

$$\longrightarrow \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \vec{0}$$

$3a + 3b = 0$, $b = -a$. So $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ works.

$\therefore$ General solution: $\boxed{\vec{x}(t) = c\begin{bmatrix} 3 \\ 2 \end{bmatrix}e^{4t} + d\begin{bmatrix} 1 \\ -1 \end{bmatrix}e^{-t}}$