Friday November 2 - Ordinary points.

So far, our algorithm is

1) Write down $y = \sum c_n x^n$, $y' = \dots$

2) Simplify until we have $\sum [\text{---}\;''\;\text{---}] x^{n+a} = 0$

3) Write & solve a recurrence relation for the coefficients.

4) Test convergence.

Really out of place! We should make sure a series converges before we find it.

An <u>ordinary point</u> of $y'' + P(x) y' + Q(x) y = 0$ is a point where P, Q are analytic.

<u>Thm.</u> Given $A(x) y'' + B(x) y' + C(x) y = 0$, then a series solution at an ordinary point for $y'' + \frac{B}{A} y' + \frac{C}{A} y = 0$ will converge, and its radius is at least the distance to the closest singular point of B/A or C/A.

<u>Ex.</u> $(x^2 + 9) y'' + (\cos x) y' + e^x y = 0$

has problems at $\pm 3i$. So we get radius 3 from origin.

We have an example in progress: $(x^2 - 4) y'' + 3x y' + y = 0$.

Our recurrence is $c_{n+2} = \dfrac{(n+1) c_n}{4(n+2)}$

Even terms: $c_{2n} = \dfrac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{4^n \cdot 2 \cdot 4 \cdot 6 \cdots (2n)} = \dfrac{(2n-1)!!}{2^{3n} n!} c_0$.