Friday 11/30 - Complex & Repeated Eigenvalues.

Complex eigenvalues of real matrices come in <u>conjugate pairs</u>.

<u>Ex.</u> $\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 5 & -9 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

Polynomial : $(5-\lambda)(-1-\lambda) - (-9)(2) = 0$

$$\lambda^2 - 4\lambda + 13 = 0$$

$$(\lambda - 2)^2 + 9 = 0 \longrightarrow \lambda = 2 \pm 3i.$$

Eigenvector for $2+3i$: $\begin{bmatrix} 3 - 3i & -9 \\ 2 & -3-3i \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(3-3i)a - 9b = 0 \longrightarrow b = \left(\frac{1}{3} - \frac{1}{3}i\right)a.$

Choosing $a = 3$ gives $\begin{bmatrix} 3 \\ 1-i \end{bmatrix}$.

The other one is $\begin{bmatrix} 3 \\ 1+i \end{bmatrix}$.

We can extract real-valued solutions by expanding

$$\begin{bmatrix} 3 \\ 1-i \end{bmatrix} e^{(2+3i)t} = \left(\begin{bmatrix} 3 \\ 1 \end{bmatrix} + i\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) e^{2t}(\cos 3t + i \sin 3t)$$

and getting the <u>real & imaginary parts</u>.

Repeated eigenvalues involve multiplication by t if there are <u>not enough eigenvectors</u>.

<u>Ex.</u> $\vec{x}\,' = \begin{bmatrix} -2 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ has eigenvalue -3, eigenvector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

We can't build two L.I. solutions, so we go back to our repeated root case. We find a second solution is $\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} t + \vec{w}\right) e^{3t}$ where $\vec{w}$ is ANY vector with $(A - (-3I))\,\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

So we need $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Could take $\vec{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, etc...