Monday 11/5 - IVPs and singular points.

Continuing last time: $(x^2 - 4)y'' + 3xy' + y = 0.$

We get $c_{2n} = \frac{(2n-1)!!}{2^{3n} n!} c_0, \quad c_{2n+1} = \frac{n!}{2^n (2n+1)!!} c_1.$

So we have $y = c_0 \left(\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^{3n} n!} x^{2n} \right) + c_1 \left(\sum_{n=0}^{\infty} \frac{n!}{2^n (2n+1)!!} x^{2n+1} \right)$

As two L.I. solutions.

Useful fact: $\boxed{y(0) = c_0. \quad y'(0) = c_1}$

That equation has bad things happen at $x = \pm 2$. Some kinds of singularities are better than others, though.

<u>Defn.</u> For the equation $x^2 y'' + x p(x) y' + q(x) y = 0$, if p, q are analytic at 0 we call 0 a <u>regular</u> singular point.

Idea: Regular = good. As a example, we had $0 = x^2 y'' + p_0 x y' + q_0 y$ that we met in <u>HW 3</u>. The solutions are x^r for r solving the <u>indicial polynomial</u> $r(r-1) + p_0 r + q_0 = 0.$

So instead of a Taylor series, look for a Frobenius series $x^r \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+r}.$