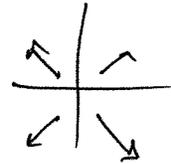


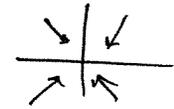
Monday 12/3 - Analysis of Systems.

The eigenvalues of a linear system determine the long-term.

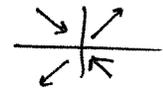
Positive eigenvalues - Source or Instability



Negative eigenvalues - Sink or stability



Different signs - Saddle



Complex: Apply the above to the real part.

If the real part is 0, get a center. Otherwise, spiral

The power of this method is to study non-linear systems. We can approximate a system with the Jacobian

where
$$\begin{aligned} x' &= f(x, y) \\ y' &= g(x, y) \end{aligned} \quad \rightsquigarrow \quad \begin{aligned} x' &\approx f_x \Delta x + f_y \Delta y \\ y' &\approx g_x \Delta x + g_y \Delta y \end{aligned} \quad \text{at critical point.}$$

Ex.
$$\begin{aligned} x' &= 2xy - 4x \\ y' &= xy - 3y \end{aligned}$$
 has critical points at $(0, 0), (3, 2)$ and Jacobian
$$\begin{bmatrix} 2y - 4 & 2x \\ y & x - 3 \end{bmatrix}.$$

At $(0, 0)$, get a stable point.

At $(3, 2)$, get a saddle.

