

Notes for 217 - Monday 8/27/18.

A differential equation is of the form

$$F(x, y, y', y'', \dots, y^{(n)}) = 0. \quad (*)$$

We say $u(x)$ is a solution on an interval I if $(*)$ holds for all $x \in I$.

Ex. (Natural growth). $y = e^{10x}$ satisfies the DE

$$y' - 10y = 0 \quad \text{because}$$

$$(e^{10x})' - 10e^{10x} = 10e^{10x} - 10e^{10x} = 0 \quad \text{for all } x \in \mathbb{R}$$

The general solution is the 1-parameter family

$$y(x) = Ce^{10x}.$$

Ex. $y(x) = \frac{1}{c-x}$ is a general solution to $dy/dx = y^2$.

If we add the initial condition $y(1) = 2$ we get the particular solution $y(x) = \frac{1}{3/2 - x}$.

Note the singularity at $x = 3/2$!!

Ex. The general solution to $y'' + y = 0$ is the two-parameter family

$$y(x) = A \cos x + B \sin x.$$

Make sure you can verify this fact.