Wednesday 8/29/18.

- Solutions via integration. If $dy/dx = f(x)$, the general solution is $y(x) = \int f(x)\,dx$; if we know $y(x_0) = y_0$, then $y(x) = \int_{x_0}^{x} f(s)\,ds + y_0$, by FTC. We can solve n^{th}-order equations too:

$$y'' = \cos x \longrightarrow y' = \sin x + C$$
$$\longrightarrow y = -\cos x + Cx + D.$$

This is great for recovering position from acceleration ($\equiv$ force).

$$y'' = -g \longrightarrow y = -\tfrac{1}{2}gt^2 + v_0 t + h_0$$

gives parabolic motion for an object in free fall.

- This technique doesn't help with (e.g.) $y' = xy$ because we cannot compute $\int y(x)\,dx$. Fix by separating variables:

$$dy/dx = xy \longrightarrow dy/y = x\,dx$$
$$\longrightarrow \int dy/y = \int x\,dx$$
$$\longrightarrow \ln|y| = \tfrac{1}{2}x^2 + C$$
$$\longrightarrow y = \pm e^{C} e^{\frac{1}{2}x^2}.$$

All x's, y's together!

Note how this misses the singular solution $y \equiv 0$.

We'll talk more about this when we see equilibria, stability, slope fields, and so on.