Friday 8/31

- Continuing separation of variables.

Ex. $2x + 2yy' = 0 \longrightarrow x^2 + y^2 = C$ gives the implicit equation of a circle. Can't always go to the explicit solution!

Ex. $dy/dx = 6e^{2x-y}$, $y(0)$ can be separated as
$$e^y\,dy = 6e^{2x}\,dx.$$

Ex. $y' = \dfrac{x^2-1}{y^4+y}$ would lead to $\frac{1}{5}y^5 + \frac{1}{2}y^2 = \frac{1}{3}x^3 - x + C$

The quintic equation may not be solvable.

Ex. $(y')^2 = 4y$, $y(0) = 0$ has ~~three~~ two solutions on $\mathbb{R}$:

$y_1(x) = x^2$, $\cancel{y_2(x) = -x^2}$, $y_3(x) = 0$.

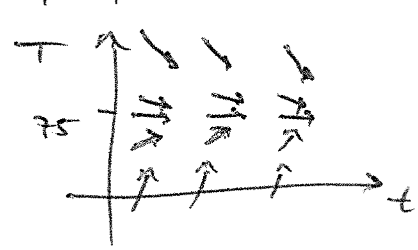
So uniqueness fails here. $\hookrightarrow$ Not a solution, sorry!

Application: We talked about Newton's Law of Cooling. If T = temperature, then $T'(t) = -k(T - \text{ambient})$. This is separable, and the solution is

$$T(t) = T_\infty + (T(0) - T_\infty)e^{-kt}$$

where T_∞ = ambient.

Can understand qualitatively with a slope field. At each point (t, T), draw an arrow with slope $T'(t)$.



Note the steady-state.