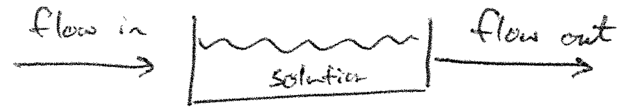
Monday 9/10 — Mixtures & Substitutions.

Typical model:



rate in r_i — rate out r_o

concentration in c_i — concentration out: unknown, equals $\frac{\text{Quantity}}{\text{Volume}}$ assuming uniform mixture.

So if Q = quantity &
V = volume,

Rate of change of Q = (Rate In) − (Rate Out)

$$\therefore \quad \frac{dQ}{dt} = r_i c_i - r_o \frac{Q}{V}$$ This is linear!

If $r_o \neq r_i$, then V depends on time too.

Substitution methods: Make things easier by "hiding" the difficulty of a problem in an ad hoc way adapted to the problem.

Ex. $y' = (x+y+2)^2 \longrightarrow v = x+y+2, \quad v' = 1 + y' \longrightarrow y' = v'-1$

Get $v' - 1 = v^2 \longrightarrow v' = 1 + v^2$ Separable!!

$\longrightarrow \arctan v = x + C$

$\longrightarrow v = \tan(x+C)$

$\longrightarrow \underline{y = \tan(x+C) - x - 2.}$

Bernoulli: $y' + P(x)y = Q(x)y^n$ Non-linear term.

Divide by y^n & let $v = y^{1-n}$, since $v' = (1-n)y^{-n}y'$.

$y^{-n}y' + Py^{1-n} = Q \longrightarrow \frac{v'}{1-n} + Pv = Q.$ Linear!