Monday 9/24 - Linear Independence.

We say $f_1, \dots, f_n$ are <u>linearly dependent</u> if there are constants $c_1, \dots, c_n$ NOT ALL ZERO such that

$$c_1 f_1 + \cdots + c_n f_n \equiv 0.$$

Otherwise, the set is <u>independent</u>.

So $\sin^2 x$, $\cos^2 x$, 1 are dependent because $\underline{1}\sin^2 x + \underline{1}\cos^2 x + \underline{(-1)}\cdot 1 = 0.$

Not all zero.

We can tell with the <u>Wronskian</u>.

$$W(f_1, \dots, f_n) = \det \begin{bmatrix} f_1 & f_2 & \cdots & f_n \\ f_1' & f_2' & \cdots & f_n' \\ & \vdots & & \\ f_n^{(n-1)} & f_2^{(n-1)} & \cdots & f_n^{(n-1)} \end{bmatrix}.$$

Compute with <u>cofactor expansion</u>.

<u>Ex.</u> $$W(1, \sin^2 x, \cos^2 x) = \det \begin{bmatrix} 1 & \sin^2 x & \cos^2 x \\ 0 & 2\sin x \cos x & -2\sin x\cos x \\ 0 & 2(\cos^2 x - \sin^2 x) & 2(\sin^2 x - \cos^2 x) \end{bmatrix}$$

$$= 1 \det \begin{bmatrix} 2\sin x \cos x & -2\sin x \cos x \\ 2(\cos^2 x - \sin^2 x) & 2(\sin^2 x - \cos^2 x) \end{bmatrix} + 0 \cdots + 0 \cdots = 0.$$

<u>Key fact.</u> $f_1, \dots, f_n$ are linearly dependent

$$\Updownarrow$$

$$W(f_1, \dots, f_n) \equiv 0.$$