Wednesday 9/26 - <u>Constant Coefficients</u>

Given $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_2 y'' + a_1 y' + a_0 y = 0$

the <u>characteristic polynomial</u> is $a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0$.

it has n roots, corresponding to n solutions.

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- If a root r is distinct, include e^{rx} in the general solution.
- If a root r occurs with multiplicity k, include $e^{rx}, xe^{rx}, x^2 e^{rx}, \ldots, x^{k-1} e^{rx}$.

Why? <u>Reduction of order</u>.

complex

- If a root is $a \pm bi$, (with conjugate root $a - bi$) include $e^{ax}\cos bx$ and $e^{ax} \sin bx$.

  Why? Euler's Identity $e^{i\theta} = \cos\theta + i \sin\theta$.

- If a complex root has multiplicity k, multiply by a polynomial with degree $k-1$.

<u>Ex.</u> $y^{(4)} + 18y'' + 81y = 0 \longrightarrow r^4 + 18r^2 + 81 = 0$

$\longrightarrow (r^2+9)^2 = 0 \longrightarrow r = \pm 3i$, mult. 2

G.S. $y = c_1 \cos 3x + c_2 \sin 3x + c_3 x \cos 3x + c_4 x \sin 3x$

<u>Ex.</u> $y^{(4)} - 4y'' = 0 \longrightarrow r^4 - 4r^2 = 0 \longrightarrow r^2(r-2)(r+2) = 0$

$\longrightarrow$ G.S. $y = c_1 + c_2 x + c_3 e^{2x} + c_4 e^{-2x}$

<u>Ex.</u> $y'' + 8y' + 25y = 0 \longrightarrow r^2 + 8r + 25 = 0 \longrightarrow (r+4)^2 + 9 = 0$

$\longrightarrow r = -4 \pm 3i$ so $y = c_1 e^{-4x} \cos 3x + c_2 e^{-4x} \sin 3x$.