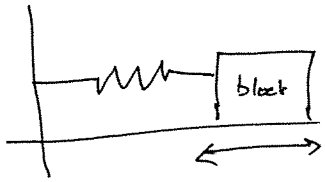
Friday 9/28 - Mechanical Vibrations.



k: spring constant from Hooke's Law

m: mass $\quad F(t)$: driving force.

Model: $m x'' + c x' + k x = F(t)$.

No damping / driving $\longrightarrow$ $x'' + \frac{k}{m} x = 0$.

Use circular frequency $\omega_0 := \sqrt{\frac{k}{m}}$ to get

$x(t) = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$.

Can rewrite as $x(t) = A \cos(\omega_0 t - \delta)$

Amplitude (A) $\quad$ time lag (δ).

with $A = \sqrt{C_1^2 + C_2^2}$ and $\tan \delta = C_2 / C_1$. Pay attention to quadrant of δ!!

Underdamped
2 complex roots

Small damping, where $p := \frac{c}{2m} < \omega_0$

$\hookrightarrow$ get $x'' + 2p x' + \omega_0^2 = 0$.

This oscillates at frequency $\sqrt{\omega_0^2 - p^2} < \omega_0$

and has decaying amplitude.

Overdamped
2 real roots

Large damping, where $p > \omega_0$ $\longrightarrow$ No oscillation.

Repeated real root.

Critical damping, where $p = \omega_0$ $\longrightarrow$ fastest return to equilibrium.