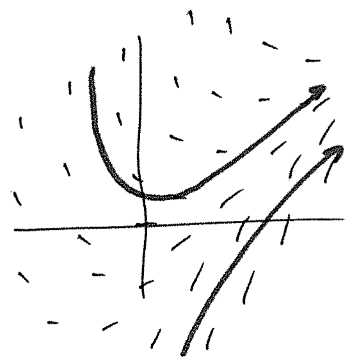
Wednesday 9/5 — Slope fields, existence - uniqueness.

We saw $y' = x - y$:

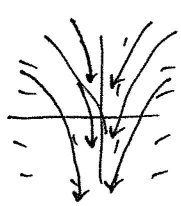


Explore plots like this on Geogebra!!

Note along $y = x - 1$, $y' = 1$. So we'd stay on this line. We get a steady growth — hypothesize our solution is $y(x) = x - 1 + \text{error}$. In fact, solution is $x - 1 + Ce^{-x}$.

Existence - Uniqueness.

$y' = \frac{1}{x}$, $y(0) = 0$ has NO SOLUTION.



$y' = 2\sqrt{y}$, $y(0) = 0$ has TWO SOLUTIONS

$y_1(x) = x^2$

$y_2(x) = 0$.

Both situations are <u>BAD</u>.

<u>Theorem</u>. $y' = f(x,y)$, $y(a) = b$ has a unique solution on an interval around a if f and $\partial_y f$ are both continuous on a rectangle containing (a,b).

<u>Ex</u>. $y' = 2\sqrt{y} \longrightarrow f(x,y) = 2\sqrt{y}$ continuous for $y \geq 0$ but $\partial_y f = \frac{1}{\sqrt{y}}$ is not continuous at $y = 0$.

<u>Ex</u>. $y' = x - y \longrightarrow f(x,y) = x - y$ and $\partial_y f = -1$ are fine <u>everywhere</u>. So get a sol'n for