Friday 9/7 - Linear Equations.

We want to solve $y' + y = x$ (not separable!).

Multiply by integrating factor e^x:

$$e^x y' + e^x y = xe^x$$

$$e^x y' + (e^x)' y = xe^x \quad \Big\downarrow \text{ By product rule.}$$

$$(e^x y)' = xe^x.$$

Now integrate to get $e^x y = xe^x - e^x + C$.

For a linear equation $y' + P(x)y = Q(x)$ we can multiply by μ to get

$$\mu y' + \mu P y = Q\mu$$

Hope: $$\mu y' + \mu' y = Q\mu$$

$$(\mu y)' = Q\mu.$$

So look for any μ with $\mu' = P\mu$, or $\boxed{\mu = e^{\int P(x)\,dx}}$.

Ex. $xy' + 2y = 0 \longrightarrow y' + \frac{2}{x} y = 0$

Use integrating factor $e^{\int 2/x\,dx} = e^{2\ln x} = x^2$.

Don't memorize the formulas! Use product rule every time, at least until you're sure how it works.