Homework 10

Math 217

Due: 7 December 2018 by 11:59 PM

Instructions: Write your solutions to the following problems and submit them on Crowdmark by the deadline. You are encouraged to work in groups or consult with each other on the problems, but the work submitted must be your own and must be written up by you.

(1) Find the (real-valued) solution to the system

\[ \begin{align*}
    x' &= 5x + 5y + 2z \\
    y' &= -6x - 6y - 5z \\
    z' &= 6x + 6y + 5z
\end{align*} \]

with initial conditions \( x(0) = 3, y(0) = 2, z(0) = 1. \)

(2) Find the general solution of

\[ \vec{x}' = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix} \vec{x}. \]

(3) Find the critical points of the system

\[ \begin{align*}
    \frac{dx}{dt} &= y - 1 \\
    \frac{dy}{dt} &= x^2 - y.
\end{align*} \]

Classify each critical point as stable/unstable/saddle, and determine whether the trajectories have any spiraling. Draw a rough phase portrait, making sure to identify the critical points and with enough detail to show the long-term behavior of the system.

(4) Carry out a careful analysis of the following two-population system. Draw a phase portrait (in the domain \( x, y \geq 0 \)), identify any equilibrium point(s) as stable/unstable/saddles, and determine under what circumstances the populations will coexist indefinitely.

\[ \begin{align*}
    \frac{dx}{dt} &= 2xy - 16x \\
    \frac{dy}{dt} &= 4y - xy.
\end{align*} \]