Instructions: Write your solutions to the following problems and submit them on Crowdmark by the deadline. You are encouraged to work in groups or consult with each other on the problems, but the work submitted must be your own and must be written up by you.

(1) Compute (from the definition) the Laplace transform of a function defined by

\[ g(t) = \begin{cases} 
  t; & 0 \leq t < 1 \\
  0; & t \geq 1 
\end{cases} \]

(2) The ceiling function is defined by \( f(t) = n \) if \( n - 1 < t \leq n \) for each integer \( n \); for example, the ceiling of 3.1 is 4, and the ceiling of 2.996 is 3. It represents a signal which jumps once per unit time by a fixed amount. The floor function can be written as

\[ f(t) = \sum_{n=0}^{\infty} u(t - n). \]

Assuming that the Laplace transform of the infinite sum is the infinite sum of the Laplace transforms (which turns out to be true), compute \( \mathcal{L}\{f(t)\} \). You may need to use the formula for the sum of a geometric series.

(3) Use the Laplace transform to solve \( x'' + 4x' + 3x = 1 \) with the initial conditions \( x(0) = x'(0) = 0 \).

(4) Use the Laplace transform to solve \( x'' + 9x = 1 \) subject to initial conditions \( x(0) = x'(0) = 0 \). Note that this corresponds to a mass-spring system where we have a constant (non-oscillatory) driving force; give a brief explanation of what the long-term oscillation looks like compared to the free spring-mass system.