Friday 10/12: Transforming ODEs.

Key fact: \[ Y \{ y'' \} = s^2 Y(s) - sy(0) - y'(0) \]

\[ Y \{ y''' \} = s^2 Y(s) - sy(0) - y'(0) \]

and so on. So we turn ODEs into purely algebraic equations.

Ex. \[ y'' - 4y' = e^t \quad y(0) = y'(0) = 0 \]

1) Translate to Laplace side

\[ s^2 Y(s) - 4sY(s) = \frac{1}{s-1} \]

\[ \rightarrow \quad Y(s) = \frac{1}{(s^2 - 4s)(s-1)} \]

2) Simplify into known form (typically by partial fractions)

\[ Y(s) = \frac{1}{s(s-1)(s-4)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-4} \]

\[ = -\frac{1}{3} \frac{1}{s-1} + \frac{1}{4} \frac{1}{s} + \frac{1}{12} \frac{1}{s-4} \]

3) Invert the transform:

\[ y(t) = -\frac{1}{3} e^t + \frac{1}{4} + \frac{1}{12} e^{4t} \]

\[ \text{particular} \quad \text{complementary} \]

Note: You will be given a table of transforms on the exams as needed – no memorizing!