Monday 10/29 - Series Solutions.

We're going to look for series solution to ODEs.

Ex. \( y' - 3y = 0 \). Let \( y = \sum_{n=0}^{\infty} c_n x^n \).

If we differentiate term-by-term, get
\[
y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n
\]

after shifting index. Then
\[
y' - 3y = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n - 3\left(\sum_{n=0}^{\infty} c_n x^n\right)
= \sum_{n=0}^{\infty} \left[(n+1) c_{n+1} - 3c_n\right] x^n = 0 ; \sum_{n=0}^{\infty} 0x^n
\]

By the identity principle, \((n+1) c_{n+1} - 3c_n = 0\) for all \(n\).

This gives a recurrence relation
\[
c_{n+1} = \frac{3c_n}{n+1}.
\]

If we solve it, \( c_n = \frac{3^n c_0}{n!} \).
\[
\therefore y = \sum_{n=0}^{\infty} \frac{3^n c_0}{n!} x^n = c_0 e^{3x}.
\]

This is a typical format. We get a relationship that determines the coefficients. Practically, only a few terms are necessary for (reasonably) high-precision computation - great for approximations!