$M \dddot{x} + C \dot{x} + kx = F(t)$: Now we can handle $F$!

Ex. $\omega_n$ damping, $F = F_0 \cos \omega t$.

\[
L \ x'' + \frac{k}{\omega_n^2} x = F_0 \cos \omega t
\]

$\omega_n^2 x = F_0 \cos \omega t$ \quad $\omega_n = \text{natural response frequency of system}$

Solve with undetermined coefficients ...

$x(t) = c_1 \cos \omega_n t + c_2 \sin \omega_n t + \frac{F_0/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \cos \omega t$

\[\text{Natural response} \quad \text{Amplified!}\]

If $\omega = \omega_n$, the response is huge and the input signal is greatly amplified. Damping decreases this:

Ex. $\dddot{x} + 2px' + \omega_n^2 x = \frac{F_0}{\omega_n} \cos \omega t$.

Underdamped case, e.g. two complex roots; solution is of the form

\[
x = c_1 (e^{-pt} \cos \omega_n t) + c_2 (e^{-pt} \sin \omega_n t) + c_3 \cos \omega n t + c_4 \sin \omega n t
\]

\[\text{transient} \to \text{amplitude} \to 0 \quad \text{steady periodic} \quad \text{due to exponentials.} \quad \text{long-term behavior.} \quad \text{Practical resonant}\]

The closer $\omega$ is to the natural frequency, the stronger the response. Applications: Radio tuners, driving on a washboard road, structural failures, etc.