Can generally solve by elimination (compare to solving algebraic systems!). Substitute to eliminate a variable.

Ex: \[ x' = y + x \]
    \[ y' = 3x - y \]

    \[ x'' = y' + x' \]
    \[ y' = 3x - (x' + y) + x' \]
    \[ = 2x. \] (from 1st equation)

Now we can get \[ x'' - 2x = 0. \]

Main goal: Translate from nth order to 1st order system.

Matrix notation is useful:

\[ \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}. \]

We can turn \[ y'' + ay' + by = f(t) \] into a system

\[ u' = v, \quad v' = u'' = -by - ay' + f = -bu - av + f. \]

We have \[ \begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} f \end{bmatrix}. \]

Why is this important? Because we can solve

\[ \dot{\mathbf{x}} = A\mathbf{x} \]
via \[ \mathbf{x} = e^{At}\mathbf{x}(0) \]
with the matrix exponential.