Monday 11/5 - IVPs and singularity points.

Continuing last time: \((x^2 - 4)y'' + 3x y' + y = 0\).

We get 
\[ c_{2n} = \frac{(2n-1)!!}{2^{3n} n!} c_0, \quad c_{2n+1} = \frac{n!}{2^n (2n+1)!!} c_1. \]

So we have 
\[ y = c_0 \left( \sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^{3n} n!} x^n \right) + c_1 \left( \sum_{n=0}^{\infty} \frac{n!}{2^n (2n+1)!!} x^{2n+1} \right) \]

as two L.I. solutions.

Useful fact: 
\[ y(0) = c_0, \quad y'(0) = c_1. \]

That equation has bad things happen at \(x = \pm 2\). Some kinds of singularities are better than others, though.

Def. For the equation \(x^n y'' + x p(x) y' + q(x) y = 0\), if \(p, q\) are analytic at 0 we call 0 a regular singular point.

Idea: Regular = good. As an example, we had 0: \(x^2 y'' + 3x y' + y = 0\), that we met in HW 3. The solutions are \(x^r\) for \( r \) solving the indicial polynomial \( r(r-1) + p_0 r + q_0 = 0 \).

So instead of a Taylor series, look for a Frobenius series \( \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+r} \).