A differential equation is of the form
\[ F(x, y, y', y'', \ldots, y^{(n)}) = 0. \]  \( \text{(1)} \)

We say \( u(x) \) is a solution on an interval \( I \) if \( (\text{1}) \) holds for all \( x \in I \).

**Ex.** (Natural growth). \( y = e^{10x} \) satisfies the DE
\[ y' - 10y = 0 \]

because
\[ (e^{10x})' - 10e^{10x} = 10e^{10x} - 10e^{10x} = 0 \text{ for all } x \in \mathbb{R} \]

The general solution is the 1-parameter family
\[ y(x) = Ce^{10x}. \]

**Ex.** \( y(x) = \frac{1}{c - x} \) is a solution to \( \frac{dy}{dx} = y^2 \),

**general**

If we add the initial condition \( y(1) = 2 \) we get the particular solution
\[ y(x) = \frac{1}{\frac{3}{2} - x}. \]

Note the singularity at \( x = \frac{3}{2} \)!!

**Ex.** The general solution to \( y'' + y = 0 \) is the two-parameter family
\[ y(x) = A \cos x + B \sin x. \]

Make sure you can verify this fact.