Wednesday 8/29/18.

- Solutions via integration. If \( \frac{dy}{dx} = f(x) \), the general solution is \( y(x) = \int f(x) \, dx \); if we know \( y(x_0) = y_0 \), then \( y(x) = \int_{x_0}^{x} f(s) \, ds + y_0 \), by FTC. We can solve \( n \)th-order equations too:

\[
\begin{align*}
y'' &= \cos x & \Rightarrow & & y' = \sin x + C \\
&\quad \Rightarrow & & y = -\cos x + Cx + D.
\end{align*}
\]

This is great for recovering position from acceleration (\( \equiv \) force),

\[
y'' = -g \Rightarrow y = -\frac{1}{2} gt^2 + v_0 t + h
\]

gives parabolic motion for an object in free-fall.

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- This technique doesn't help with (e.g.) \( y' = xy \),

because we cannot compute \( \int y(x) \, dx \). Fix by **separating variables**:

\[
\begin{align*}
dy/dx &= xy & \Rightarrow & & dy/y = x \, dx \\
&\quad \Rightarrow & & \int dy/y = \int x \, dx \\
&\quad \Rightarrow & & \ln |y| = \frac{1}{2} x^2 + C \\
&\quad \Rightarrow & & y = \pm Ce^{\frac{1}{2} x^2}.
\end{align*}
\]

Note how this misses the singular solution \( y = 0 \). We'll talk more about this when we see equilibria, stability, slope fields, and so on.