Monday 9/10 - Mixtures & Substitutions.

Typical model:

\[
\text{flow in} \quad \text{flow out}
\]

\[
solution
\]

rate in \( r_i \)
concentration in \( c_i \)
rate out \( r_o \)
concentration out \( c_o \)

So if \( Q = \text{quantity} \)
\( V = \text{volume}, \)

Rate of change of \( Q \) = (Rate in) - (Rate Out)

\[
\frac{dQ}{dt} = r_i c_i - r_o \frac{Q}{V}
\]

This is linear!

If \( r_o \neq r_i \), then \( V \) depends on time too.

Substitution methods: Make things easier by "hiding" the difficulty of a problem in an ad hoc way adapted to the problem.

\[ y' = (x + y + 2)^2 \rightarrow v = x + y + 2 \quad \text{and} \quad v' = 1 + y' \rightarrow y's = v' \]

Get \( v' - 1 = v^2 \rightarrow v' = 1 + v^2 \) Separable\!

\[
\arctan v = x + C
\]

\[
v = \tan (x + C)
\]

\[
y = \tan (x + C) - x - 2.
\]

Bernoulli: \( y' + Py = Q(x)y^m \) Non-linear term.

Divide by \( y^m \) & let \( v = y^{m-1} \), since \( v' = (m-1)y^{-m}y' \).

\[
y^{-m}y' + P y^{m-1} = Q \rightarrow \frac{v'}{1-m} + Pv = Q \quad \text{Linear!}
\]