Homework 10

Math 308

Due: 26 April

Guidelines:

- You are strongly encouraged to work together to understand the problems, but what you turn in must be your own work.
- Your submission must be clearly written and stapled. Homework will only be accepted up to the beginning of lecture, or you can drop it off at my office before class.
- You must **cite all your sources**, or say that your submission did not use any other resources. If you use a symbolic calculator, reference text, formula sheet, internet resource, or anything similar, it must be clearly stated in your work where and what you used. **A problem without citations will be considered incomplete.**

(1) (9.5.7) If a particle is constrained to stay on a surface defined implicitly by $f(x, y, z) = 0$, and there are no other forces on the particle, show that it will move along a geodesic.

(2) (9.5.13) A particle moves along the surface of a paraboloid $z = x^2 + y^2$; the only force acting on it is gravity, and there is no friction. Find the Lagrangian and the equations of motion, and show that the particle can move in a horizontal circle.

(3) (9.6.1) Given the length ℓ of a curve joining two points, find the equation of the curve that minimizes the area of the surface of revolution formed by rotating the curve about the x-axis.

(4) (9.6.2) Given the length ℓ of a curve joining two points, find the equation of the curve that maximizes the area between the curve and the straight line joining the points.

(5) (9.8.24) Write and simplify the Euler-Lagrange equation to make the integral

$$\int_a^b [P(x, y) + Q(x, y)y']dx$$

stationary, and show that if the equation is satisfied then the integral has the same value for all paths joining a to b. Comment on the relationship with Green's theorem.

(6) (9.8.28) Write the θ Euler-Lagrange equation for a particle moving in a plane subject to a radial potential (so that $V(r, \theta)$ only depends on r). Use it to show that

(a) Angular momentum is conserved.

(b) The position $\vec{r}$ sweeps out equal areas in equal time.