Homework 7

Math 308

Due: 25 March

Guidelines:

- You are strongly encouraged to work together to understand the problems, but what you turn in must be your own work. If you collaborate with anyone else on this homework, you must write the names of your collaborators on your submission.
- You must cite all your sources, or say that your submission only involves your own work. If you use a calculator, reference text, formula sheet, internet resource, etc. you must clearly identify where and how you have used it. Every problem must have either a full set of citations, or a statement that you did not use any other resources.
- Your submission must be clearly written and stapled. Homework will only be accepted up to the beginning of lecture, or you can drop it off at my office before class.

(1) A particle with mass m undergoing simple harmonic motion with amplitude A has its displacement given by $y = A \sin \omega t$. Compute the average kinetic energy of the particle over a full period of oscillation and show that it is proportional to the square of the amplitude. Recall that the energy at time t is given by $\frac{1}{2}m\dot{y}^2$.

(2) (7.13.13) Find the Fourier series with period 2 for the function $f(x) = (x-1)^2$ on $(0, 2)$ and use it to evalute $\sum_{n=1}^{\infty} 1/n^4$.

(3) Consider a square wave whose voltage is $V(t) = 100$ for $0 < t < 1/120$ and $V(t) = -100$ for $1/120 < t < 1/60$, and the signal repeats 60 times per second.

(a) Expand this signal in an appropriate Fourier series and identify which frequencies are most apparent.

(b) Estimate how much (relative) energy is lost if we replace the signal by using the first five frequencies in the sine/cosine series.

(c) Estimate how much (relative) energy is lost if we replace the signal by using only the five most intense frequencies.

(d) Plot the wave and its approximations from parts (b) and (c) together.

(4) Consider a rectified half-wave; the curve is given by a sine function for half the cycle and zero for the other half (see the figure in problem 7.10.5), and the signal repeats 60 times per second. Repeat problem 3 for this signal.

(5) A string of length l has zero initial vlocity and its displacement is triangular with height h above the left quarter of the string (see the diagram in 13.4.3). Its ends are fixed for all time. Find its displacement as a function of x and t. Identify the most important harmonic(s).

(6) A string of length l is stretched straight and its ends are fixed for all time. The string is hit with a hammer and the initial velocity imparted is shaped like a triangle with vertices at the endpoints and at $(l/3, h)$ (see the diagram in 13.4.7). Find its displacement as a function of x and t. Identify the most important harmonic(s).