

# Homework 9

Math 308

Due: 15 April

## Guidelines:

- You are strongly encouraged to work together to understand the problems, but what you turn in must be your own work.
  - Your submission must be clearly written and stapled. Homework will only be accepted up to the beginning of lecture, or you can drop it off at my office before class.
  - You must **cite all your sources**, or say that your submission did not use any other resources. If you use a symbolic calculator, reference text, formula sheet, internet resource, or anything similar, it must be clearly stated in your work where and what you used. **A problem without citations will be considered incomplete.**
- (1) (9.1.1) If light passes through some medium, then Fermat's principle states that the path will minimize transit time. In a setting with constant index of refraction, light moves at a constant speed. Use this fact to derive the optical law of *reflection*, namely that the incident angle of light on a surface is the same as the reflected angle. Start by considering light moving from a point  $A$  to a point  $B$  and hitting the mirror at position  $P$ . See the diagram on p. 474.
  - (2) Suppose that the index of refraction of a material is  $\sqrt{x}$  so that the velocity of the light is  $c/\sqrt{x}$  (normalized so that  $c = 1$ ). The transit time of the light from some point to another is then  $\int_{x_1}^{x_2} \sqrt{x} ds$ . Write and solve the corresponding Euler-Lagrange equation to make this integral stationary, and show that the light moves in a parabolic trajectory.
  - (3) (9.2.9) Write and solve the Euler-Lagrange equations to make  $I = \int_{x_1}^{x_2} \sqrt{1 + y^2(y')^2} dx$  stationary; make a change of variable if needed.
  - (4) (9.3.15) Find the geodesics on a plane using polar coordinates.
  - (5) (9.3.18) Find the geodesics on a sphere.

**Suggested problems:** These problems are not to be submitted, but I strongly encourage you to work through them in full detail.

- 9.1.2, the derivation of Snell's law. Very similar to 9.1.1.
- 9.3.14, path of a light ray whose velocity grows radially.
- 9.3.16, geodesics on a cylinder.
- Taxes. This isn't for credit from me, but do it anyways.