1. In stretching a spring 3 inches beyond its natural length, the work done was 60 ft-lb. What was the maximum force exerted?

Solution
2. A cable hanging over the side of a 100 ft building is used to lift a 200 lb load 40 ft up from the ground. The cable weighs 5 lb per linear foot. How much work was done?

Solution

There are three components to the total work: the work done on the load, the done on the lowest 60 feet of cable, and the work done on the highest 40 feet of cable. Calculus is needed only for the third of these components.

\[
\text{Work} = 200 \times \text{lb} \times 40 \times \text{ft} + (60 \times \text{ft} \times 5 \times \text{lb/ft}) \times 40 \times \text{ft} + \int (y \times 5 \times \text{lb/ft}, y = 0 \times \text{ft} \ldots 40 \times \text{ft});
\]

\[
\text{Work} = 24000 \times \text{ft} \times \text{lb}
\]
3. A container has the shape obtained by rotating \( y = \frac{x^4}{16} \), \( 0 \leq x \leq 4 \) ft about the y-axis. It is partially filled with water to a depth of 12 ft. If water is pumped over the top of the container until 2 ft of water remains in the container, how much work has been done? Use 62 lb per cubic ft for the weight density. Leave the integral UNEVALUATED.

**Solution**

The height of the container is \( \frac{4^4}{16} \) or 16 ft. A slice at level \( y \) and thickness \( dy \) must be raised \( 16 - y \) ft. The volume of that slice is \( \pi \left( 16y \right)^2 \) \( dy \) cubic ft, or \( 4 \pi \sqrt{y} \) \( dy \) ft\(^3\). The weight of that slice is \( 4 \pi \sqrt{y} \) dy lb. The work done raising that slice \( 16 - y \) ft is \( 4 \pi \left( 16 - y \right) \sqrt{y} \) dy ft lb. The total work done is

\[
\int_{2}^{12} 4 \pi \left( 16 - y \right) \sqrt{y} \ dy \ ft \ lb
\]

4. A 6 ft long trough (not a cone, inverted or otherwise) has a trapezoidal cross-section that looks like
The trough is 4 ft high, 4 ft wide at its base, and 6 ft wide at its top. It is filled to the top with water weighing 62 lb per cubic ft. How much work is done completely emptying the trough? Leave the integral \textit{UNEVALUATED}.

Solution

Positive y-Axis

Oriented Downward Setup
In the figure above, the positive y-axis is oriented downwards with $y = 0$ at the top of the trough. That is, the top of the trough lies on the x-axis. The points below the top have positive $y$-coordinates.

The volume of a slice at level $y$ is

$$6 \left( 2 \left( 3 - \frac{y}{4} \right) \right) \, dy$$ cubic feet.

The weight of the slice is

$$62 \left( 6 \left( 2 \left( 3 - \frac{y}{4} \right) \right) \right) \, dy$$ cubic feet.

The work done is
\[ \int_{0}^{4} 62 \cdot 6 \cdot 2 \left( 3 - \frac{y}{4} \right) y \, dy \quad \text{foot pounds.} \]

> \texttt{Int(62*6*2*(3-y/4)*y, y = 0 .. 4) = int(62*6*2*(3-y/4)*y, y = 0 .. 4);} \\
# However, an alternative setup is about to be done. 
# Evaluations of the two approaches will be compared to ensure 
# that the two integrals lead to the same value of work.

\[ \int_{0}^{4} 744 \left( 3 - \frac{y}{4} \right) y \, dy = 13888 \]

**Alternative Solution**

**Positive y-Axis Oriented Upward**

**Setup**
In the figure above, the positive y-axis is oriented upwards with \( y = 0 \) at the bottom of the trough. That is, the bottom of the trough lies on the x-axis. The volume of a slice at level \( y \) is

\[
6 \left( 2 \left( 2 + \frac{y}{4} \right) \right) dy \text{ cubic feet.}
\]

The weight of the slice is

\[
62 \left( 6 \right) \left( 2 \left( 2 + \frac{y}{4} \right) \right) dy \text{ cubic feet.}
\]

This slice must be raised a distance \( 4 - y \).
The work done is

\[\int_0^4 62 \cdot 6 \cdot 2 \left( 2 + \frac{y}{4} \right) (4 - y) \, dy \text{ foot pounds.}\]

\[\text{Int} \left( 62 \cdot 6 \cdot 2 \cdot (2 + y/4) \cdot (4 - y), y = 0 \ldots 4 \right) =\]

\[\int_0^4 744 \left( 2 + \frac{y}{4} \right) (4 - y) \, dy = 13888\]

5. Calculate \(\int_1^4 \sqrt{x} \ln(x) \, dx\).

**Solution**

\[\text{Int} (\sqrt{x} \cdot \ln(x), x=1..4) = \text{intparts}(\]
\[\text{Int} (\sqrt{x} \cdot \ln(x), x=1..4), \ln(x));\]
\[#\]
\[#\text{inegration by parts with } u = \ln(x), dv = \sqrt{x} \, dx, du = (1/x) \, dx,\text{ and } v = (2/3)x^{3/2}\]
\[\int_1^4 \sqrt{x} \ln(x) \, dx = \frac{8}{3} \sqrt{4 \ln(4)} - \int_1^4 \frac{2 \sqrt{x}}{3} \, dx\]

\[\text{Int} (\sqrt{x} \cdot \ln(x), x=1..4) = \text{simplify} \left( 8/3 \cdot 4^{1/2} \cdot \ln(4) \right) -\]
\[\text{value} \left( \text{Int} (2/3 \cdot x^{1/2}, x = 1 \ldots 4) \right);\]
\[\int_1^4 \sqrt{x} \ln(x) \, dx = \frac{32}{3} \ln(2) - \frac{28}{9}\]
\[
\int_1^4 \sqrt{x} \ln(x) \, dx = \frac{32}{3} \ln(2) - \frac{28}{9}
\]

6. Calculate \( \int_0^\pi x \cos(x) \, dx \).

**Solution**

\[
\text{with(student):}
\]
\[
\text{eqn1 := J = Int(x*cos(x), x = 0 .. Pi/4);}
\]
\[
\#
\]
\[
\# \text{Call the value we seek J for easy reference}
\]
\[
eqn1 := J = \int_0^{\pi/4} x \cos(x) \, dx
\]
\[
\text{eqn2 := J = intparts(rhs(eqn1), x);}
\]
\[
\#
\]
\[
\# \text{Here we integrate by parts with } u = x, \ dv = \cos(x) \, dx.
\]
\[
eqn2 := J = \frac{\pi \sqrt{2}}{8} - \int_0^{\pi/4} \sin(x) \, dx
\]
\[
\text{eqn3 := lhs(eqn2) = value(rhs(eqn2));}
\]
\[
\#
\]
\[
\# \text{Here we mop up, evaluating } \int \sin(x), \ x = 0 .. \pi/4
\]
\[
eqn3 := J = \frac{\pi \sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1
\]

As a one-line verification, we will use Maple's built-in integrator:
\[
\int_0^\frac{\pi}{4} x \cos(x) \, dx = \frac{\pi \sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1
\]

7. Calculate \( \int x^2 e^x \, dx \).

Solution

\[
\int(x^2 \cdot \exp(x), x) = \text{intparts}(\int(x^2 \cdot \exp(x), x), x^2);
\]

\# Integrate by parts with \( u = x^2 \), \( dv = \exp(x) \, dx \), \( du = 2x \, dx \), \( v = \exp(x) \)

\[
\int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx
\]

\[
\int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx + \int e^x \, dx
\]

\[
\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2 \int e^x \, dx
\]

\[
\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2 e^x + C
\]

\[
\int x^2 e^x \, dx = \text{factor}(x^2 \cdot \exp(x) - 2x \cdot \exp(x) + 2 \cdot \exp(x)) + C;
\]

\# Factor

\[
\int x^2 e^x \, dx = e^x (x^2 - 2x + 2) + C
\]

\[
\int(x^2 \cdot \exp(x), x) = \text{int}(x^2 \cdot \exp(x), x) + C;
\]

\#
8. Evaluate \( \int \sin(x)^2 \, dx \).

**Solution**

\[
\int \sin(x)^2 \, dx = \int \left( \frac{1 - \cos(2x)}{2} \right) \, dx = \frac{1}{2} - \frac{1}{2} \cos(2x) + C
\]

9. Calculate constants A and B such that

\[
\int_{0}^{\frac{\pi}{2}} \sin(x)^2 \cos(x)^4 \, dx = A + B \int_{0}^{\frac{\pi}{2}} \cos(x)^4 \, dx.
\]

**Solution**

\[
\int_{0}^{\frac{\pi}{2}} \sin(x)^2 \cos(x)^4 \, dx = \int (1 - \cos(x)^2) \cos(x)^4 \, dx
\]
\[
= \int \cos(x)^4 \, dx - \int \cos(x)^6 \, dx
\]
\[
\begin{align*}
\int \sin(x)^2 \cos(x)^4 \, dx &= \int \cos(x)^4 \, dx - \left( \frac{\sin(x) \cos(x)^5}{6} + \frac{5}{6} \int \cos(x)^4 \, dx \right) \\
\int \sin(x)^2 \cos(x)^4 \, dx &= -\frac{\sin(x) \cos(x)^5}{6} + \frac{1}{6} \int \cos(x)^4 \, dx
\end{align*}
\]

The expression on the rhs evaluates to 0 at both \( x = 0 \) and \( x = \frac{\pi}{2} \).

Therefore,
\[
\int_0^{\frac{\pi}{2}} \sin(x)^2 \cos(x)^4 \, dx = A + B \int_0^{\frac{\pi}{2}} \cos(x)^4 \, dx
\]

with \( A = 0 \) and \( B = \frac{1}{6} \).

Verication using Maple's built-in integrator:

\[
\begin{align*}
&> \text{int(} \sin(x)^2 \cos(x)^4, x = 0 .. \pi/2) ; \\
&\quad (1/6) * \text{int(} \cos(x)^4, x = 0 .. \pi/2) ; \\
&\quad \int_{0}^{\pi/2} \cos(x)^4 \, dx = \frac{\pi}{32} \\
&\quad \int_{0}^{\pi/2} \cos(x)^4 \, dx = \frac{\pi}{32}
\end{align*}
\]

10. Evaluate \( \int \sin(x)^2 \cos(x)^3 \, dx \).

\textbf{Solution}

\[
\begin{align*}
&> \text{eqn1 := Int(} \sin(x)^2 \cos(x)^3, x) = \\
&\quad \text{Int(} \sin(x)^2 \text{*(}1-\sin(x)^2\text{)*} \cos(x), x) ; \\
&\# \\
&\quad \# \text{ Write } \cos(x)^3 \text{ as } \cos(x)^2 \text{*} \cos(x), \text{ or } (1 - \sin(x)^2) \cos(x)
\end{align*}
\]
11. Make a trigonometric substitution to express the integral
\[ \int_{\sqrt{2}}^{\sqrt{3}} \frac{(4-x^2)^{3/2}}{x} \, dx \] in the form \( A \int_{\alpha}^{\beta} \sin^p(\theta) \cos^q(\theta) \, d\theta \).

(Your answer should identify the values of \( A, p, q, \alpha, \) and \( \beta \).)

**Solution**

\[ > \text{eqn1 := } J = \text{Int}((4-x^2)^{3/2}/x, x = \text{sqrt}(2) .. \text{sqrt}(3)); \]

\[ > \text{eqn2 := lhs(eqn1) = changevar( u = sin(x), rhs(eqn1), u); } \]

\[ > \text{eqn3 := lhs(eqn2) = Int(expand(integrand(rhs(eqn2))), u); } \]

\[ > \text{eqn4 := lhs(eqn3) = value(rhs(eqn3)) + C; } \]

\[ > \text{eqn5 := lhs(eqn4) = subs( u = sin(x), rhs(eqn4) ); } \]
# We need express J as an integral with integrand $\sin^p * \cos^q$

\[ eqn1 := J = \int \frac{\sqrt{3}(4-x^2)^{(3/2)}}{x} \, dx \]

\[ eqn2 := \text{lhs}(eqn1) = \text{changevar}(x = 2\sin(\theta), \text{rhs}(eqn1), \theta); \]

# Make the substitution $x = 2\sin(\theta)$, $dx = 2\cos(\theta) \, d\theta$

\[ eqn2 := J = \int \frac{(4-4\sin^2(\theta))^{(3/2)}}{\sin(\theta)} \, d\theta \]

\[ eqn3 := \text{lhs}(eqn2) = \text{subs}(4-4\sin^2(\theta)^2)^{(3/2)} = 8\cos^3(\theta), \text{rhs}(eqn2) ); \]

\[ eqn3 := J = \int \frac{8\cos^4(\theta)}{\sin(\theta)} \, d\theta \]

So, $A = 8$, $p = -1$, $q = 4$, $\alpha = \frac{\pi}{4}$, and $\beta = \frac{\pi}{3}$.

Verification:

\[ \text{value}(\text{rhs}(eqn1)); \]

\[ \text{value}(\text{rhs}(eqn3)); \]

\[
\begin{align*}
81 \frac{\pi}{16} \\
\end{align*}
\]

12. Make a trigonometric substitution to express the integral
\[
\int_{0}^{4} \frac{1}{\sqrt{9 + x^2}} \, dx \quad \text{in the form} \quad A \int_{\alpha}^{\beta} \sin(\theta)^p \cos(\theta)^q \, d\theta.
\]

(Your answer should identify the values of \( A, p, q, \alpha, \) and \( \beta \).)

**Solution**

\[
> \text{eqn1 := } J = \text{Int}(1/(9+x^2)^{(1/2)}, \, x=0..4);
\]

\[
eqn1 := J = \int_{0}^{4} \frac{1}{\sqrt{9 + x^2}} \, dx
\]

\[
> \text{eqn2 := } J = \text{changevar}(x=3*tan(theta), \, \text{rhs(eqn1)}, \, \theta);
\]

\[
\begin{align*}
\# & \quad \text{Make the change of variable } x=3*tan(theta), \, dx = 3*sec(theta)^2*dtheta \\
\end{align*}
\]

\[
eqn2 := J = \int_{0}^{\text{arctan}(4/3)} \frac{3 + 3 \tan(\theta)^2}{\sqrt{9 + 9 \tan(\theta)^2}} \, d\theta
\]

\[
> \text{eqn3 := } J = \text{subs}( \{3+3*tan(theta)^2 = 3*sec(theta)^2, \, 9+9*tan(theta)^2 = 9*sec(theta)^2\}, \, \text{rhs(eqn2)});
\]

\[
eqn3 := J = \int_{0}^{\text{arctan}(4/3)} \frac{1}{3} \sqrt{9 \sec(\theta)^2} \, d\theta
\]

\[
> \text{eqn4 := } J = \text{Int( simplify(}
\begin{align*}
& \quad \text{convert(integrand(rhs(eqn3)),sincos), assume=positive \),} \\
& \quad \theta=0..\text{arctan}(4/3));
\end{align*}
\]

\[
eqn4 := J = \int_{0}^{\text{arctan}(4/3)} \frac{1}{\cos(\theta)} \, d\theta
\]

So, \( A = 1, p = 0, \, q = -1, \quad \alpha = 0, \) and \( \beta = \text{arctan}\left(\frac{4}{3}\right) \).

The next line uses Maple’s built-in numeric integrator to verify that the last integral does equal the given integral.
13. Express the integral
\[ \int \frac{(x^2 - 4)^{3/2}}{x} \, dx \]
in the form
\[ A \int \sin(\theta)^p \cos(\theta)^q \, d\theta. \]
(Your answer should identify the values of \( A, p, \) and \( q \).)

**Solution**

\[ \text{eqn1 := } J = \text{Int}((x^2-4)^{3/2}/x, x); \]
\[ \text{eqn1 := } J = \int \frac{(x^2 - 4)^{3/2}}{x} \, dx \]
\[ \text{eqn2 := } J = \text{changevar}(x=2 \cdot \text{sec}(\theta), \text{rhs(eqn1)}, \theta); \]
\[ \text{eqn2 := } J = \text{changevar} \left( x = 2 \cdot \text{sec}(\theta), \frac{(x^2 - 4)^{3/2}}{x}, \theta \right) \]
\[ \text{eqn3 := } J = \text{subs}(4 \cdot \text{sec}(\theta)^2 - 4 = 4 \cdot \text{tan}(\theta)^2, \text{rhs(eqn2)}); \]
\[ \text{eqn3 := } J = \text{changevar} \left( x = 2 \cdot \text{sec}(\theta), \frac{(x^2 - 4)^{3/2}}{x}, \theta \right) \]
\[ \text{eqn4 := } J = 8 \cdot \text{Int}(\text{tan}(\theta)^4, \theta); \]
\[ \text{eqn4 := } J = 8 \int \text{tan}(\theta)^4 \, d\theta \]
\[ \text{eqn5 := } J = \text{map}(z \rightarrow \text{convert}(z, \text{sincos}), \text{rhs(eqn4)}); \]
\[ \text{eqn5} := J = 8 \int \frac{\sin(\theta)^4}{\cos(\theta)^4} d\theta \]

\[ > \text{A = 8, p = 4, q = -4;} \]

In the next line, we ask Maple to evaluate the given indefinite integral:

\[ > \text{eqn6} := \text{Int}((x^2-4)^{(3/2)}/x,x) = \text{int}((x^2-4)^{(3/2)}/x,x); \]

\[ \text{eqn6} := \int (x^2-4)^{3/2} \frac{dx}{x} = \frac{(x^2-4)^{3/2}}{3} - 4 \sqrt{x^2-4} - 8 \arctan \left( \frac{2}{\sqrt{x^2-4}} \right) \]

Now let us ask Maple to evaluate the transformed indefinite integral that we found:

\[ > \text{eqn7} := \text{Int}((x^2-4)^{(3/2)}/x,x) = \text{value}(\text{rhs(eqn5)}); \]

\[ \text{eqn7} := \int (x^2-4)^{3/2} \frac{dx}{x} = \frac{8}{3} \tan(\theta)^3 - 8 \tan(\theta) + 8 \theta \]

\[ > \text{eqn8} := \text{lhs(eqn7)} = \text{subs} \{ \text{tan(\theta)} = \sqrt{x^2-4}/2, \theta = \text{arctan}(\sqrt{x^2-4}/2) \}, \text{rhs(eqn7)}; \]

\[ \# \]

\[ \# \text{We resubstitute to recover the original variable } x \]

\[ \text{eqn8} := \int (x^2-4)^{3/2} \frac{dx}{x} = \frac{(x^2-4)^{3/2}}{3} - 4 \sqrt{x^2-4} + 8 \arctan \left( \frac{\sqrt{x^2-4}}{2} \right) \]

This NOT the same: compare the arctangent terms.

\[ > \text{difference} := \text{simplify}(\text{rhs(eqn6)} - \text{rhs(eqn8)}); \]

\[ \text{difference} := -8 \arctan \left( \frac{2}{\sqrt{x^2-4}} \right) - 8 \arctan \left( \frac{\sqrt{x^2-4}}{2} \right) \]
However, the two antiderivatives do differ by a constant. See what happens when two different values of $x$ are substituted:

\[
> \text{evalf( subs(x = 5.12346789, difference) )}; \\
-12.56637062 \\
> \text{evalf( subs(x = 9.87643215, difference) )}; \\
-12.56637061
\]

OK: A small rounding discrepancy.

We can check the assertion for all values of $x$ by differentiating. The derivative of the rhs of eqn8 should equal the integrand of the lhs. In other words, the difference should be 0.

\[
> \text{simplify(integrand(lhs(eqn8)) - diff(rhs(eqn8), x))}; \\
0
\]

A plot can also be used to provide visual evidence that the difference is a constant:
14. What is the partial fraction decomposition of \( \frac{2x - 1}{x^2 + 5x + 4} \)?

**Solution**

```maple
> convert((2*x-1)/(x^2+5*x+4), parfrac, x);
```

\[
\frac{3}{x + 4} - \frac{1}{x + 1}
\]

15. What is the partial fraction decomposition of \( \frac{x^3 + 2x^2 + 2x + 4}{x^2(x^2 + x + 2)} \)?

**Solution**

Maple answers this question with a one-line command:

```maple
> convert((x^3+2*x^2+2*x+4)/(x^2)*(x^2+x+2), parfrac, x);
```

Here are the details:

```maple
> eqn1 := (x^3+2*x^2+3*x+4)/(x^2)/(x^2+x+1) = A/x + B/x^2 + (C*x+E)/(x^2 + x + 1);
```

Here is the form of the partial fraction expansion:

\[
\frac{x^3 + 2x^2 + 3x + 4}{x^2(x^2 + x + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C x + E}{x^2 + x + 1}
\]

```maple
> eqn2 := lhs(eqn1) = normal(rhs(eqn1));
```

#
> eqn2 := \frac{x^3 + 2x^2 + 3x + 4}{x^2(x^2 + x + 1)} = \frac{A x^3 + A x^2 + A x + B x^2 + B x + B + x^3 C + x^2 E}{x^2(x^2 + x + 1)}

> eqn3 := numer(lhs(eqn2)) = numer(rhs(eqn2));

# Equates numerators of equal fractions, given that they have the same denominators

> eqn4 := subs(x = 0, eqn3);

# The original denominator has one real root, 0. May as well substitute it to pick off one coefficient immediately

> eqn5 := subs(B = 4, eqn3);

# Now that the value of B is known, substitute it back into the main identity, eqn3

> eqn6 := lhs(eqn5) = collect(rhs(eqn5), x);

# Expand and collect terms on the right side of eqn5

> solve( {A+C = 1, (4+A+E) = 2, 4+A = 3}, {A,C,E} );

# Equate coefficients of like powers of x on both sides of eqn6. Then solve the simultaneous equations.

{A = -1, E = -1, C = 2}

16. Calculate \[ \int \frac{4x + 3}{x^2 + 2x + 5} dx. \]

Solution

> with(student):

> eqn1 := Int((4*x+3)/(x^2+2*x+5), x) =
\[
\text{Int}\left(\frac{4x+3}{\text{completesquare}(x^2+2x+5,x)}\right)\,dx = \int \frac{4x+3}{(x+1)^2+4} \,dx
\]

\text{eqn1} := \int \frac{4x+3}{x^2+2x+5} \,dx = \int \frac{4x+3}{(x+1)^2+4} \,du

> eqn2 := u = x + 1;

\text{eqn2} := u = x + 1

> eqn3 := \text{lhs(eqn1)} = \text{changevar(eqn2, rhs(eqn1), u)};

\text{eqn3} := \int \frac{4x+3}{x^2+2x+5} \,dx = \int \frac{-1+4u}{u^2+4} \,du

> eqn4 := \text{lhs(eqn3)} = \text{expand(rhs(eqn3))};

\text{eqn4} := \int \frac{4x+3}{x^2+2x+5} \,dx = -\int \frac{1}{u^2+4} \,du + \frac{u}{u^2+4} \,du

> eqn5 := \text{lhs(eqn4)} = \text{value(op(rhs(eqn4))[1]) + op(rhs(eqn4))[2]};

\text{eqn5} := \int \frac{4x+3}{x^2+2x+5} \,dx = -\frac{1}{2} \text{arctan}\left(\frac{u}{2}\right) + 4 \int \frac{u}{u^2+4} \,du

> eqn6 := w = u^2 + 4;

\text{eqn6} := w = u^2 + 4

> eqn7 := \text{lhs(eqn5)} = \text{op(rhs(eqn5))[1]} + \text{changevar(eqn6, op(rhs(eqn5))[2], w)};

\text{eqn7} := \int \frac{4x+3}{x^2+2x+5} \,dx = \frac{1}{2} \text{arctan}\left(\frac{u}{2}\right) + 4 \int \frac{1}{2w} \,dw

> eqn8 := \text{lhs(eqn7)} = \text{op(rhs(eqn7))[1]} + \text{value(op(rhs(eqn7))[2])} + C;

\text{eqn8} := \int \frac{4x+3}{x^2+2x+5} \,dx = \frac{1}{2} \text{arctan}\left(\frac{u}{2}\right) + 2 \text{ln}(w) + C

> eqn9 := \text{lhs(eqn8)} = \text{subs(eqn6, rhs(eqn8))};

\text{eqn9} := \int \frac{4x+3}{x^2+2x+5} \,dx = \frac{1}{2} \text{arctan}\left(\frac{u}{2}\right) + 2 \text{ln}(u^2+4) + C

> eqn10 := \text{lhs(eqn9)} = \text{subs(eqn2, rhs(eqn9))};

\text{eqn10} := \int \frac{4x+3}{x^2+2x+5} \,dx = \frac{1}{2} \text{arctan}\left(\frac{x+1}{2}\right) + 2 \text{ln}((x+1)^2+4) + C

> testeq( \text{integrand(lhs(eqn10))} = \text{diff(rhs(eqn10), x)});

# Verification: The derivative of the antiderivative we found
17. Use the Trapezoidal Rule to estimate the area between the two curves graphed below.

Solution

\[
N := 6: \\
\text{Delta}_x := (6-0)/6; \\
T[6] := (\text{Delta}_x/2)*(1*0 + 2*4 + 2*4 + 2*4 + 2*6 + 2*6 + 1*0); \\
\]

\[
\text{Delta}_x := 1 \\
T_6 := 24
\]

18. Redo the estimate of the preceding problem using Simpson's Rule.
Solution

\[ N := 6; \]
\[ \Delta x := (6-0)/6; \]
\[ \Delta x := 1 \]
\[ S[6] := (\Delta x/3)*(1*0 + 4*4 + 2*4 + 4*4 + 2*6 + 4*6 + 1*0); \]
\[ \text{evalf}(S[6]); \]
\[ S_6 := \frac{76}{3} \]
\[ 25.33333333 \]
\[ Si[6] := 76/3; \]