Practice Exam 2

Math 309

November 9, 2016

You may use a scientific calculator, but no notes, graphing calculators, or other electronic devices allowed.

Fill out your scantron cards with your name, *including your official "preferred" name*, and your ID number.

Part I: Multiple Choice (30) Each problem in this section is worth five points.

1. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T(\mathbf{x}) = A\mathbf{x}$ for a certain matrix A. If S a two by three rectangle, the image under the transformation $T(S)$ is a parallelogram with area 36. Which of the following matrices might be A?

   A) $\begin{bmatrix} 17 & 31 \\ -12 & 22 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 0 \\ -12 & -3 \end{bmatrix}$

   E) $\begin{bmatrix} 7 & 3 \\ -3 & 0 \end{bmatrix}$ F) $\begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$ G) $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ H) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

2. Find all x such that $\det \begin{bmatrix} 0 & 0 & 2x \\ 10 & x & -4 \\ -3 & 0 & 36 \end{bmatrix} = \det \begin{bmatrix} x & -2 & 11 \\ 0 & 5 & -2 \\ 0 & -4 & 4 \end{bmatrix}$

   A) all real numbers x B) no real numbers x C) 0, -2 D) 0, 2
   E) 6, 12 F) 0, 6 G) -2, 6 H) none of the above

3. Let A be an $m \times n$ matrix and suppose the reduced echelon form of A has two rows of zeros. What is the dimension of Nul(A)?

   A) m B) n C) n - m D) n - m + 2 E) m - n + 2
   F) m + n G) m + n +2 H) none of the above

4. If A and B are 2×2 matrices, $\det(A) = 4$ and $\det(B) = 3$, what is $\det((3A)B^{-1})$?

A) 1 B) -1 C) 4 D) 12 E) -12 F) 54 G) -108 H) none of the above

5. If H and K are subspaces of a finite dimensional vector space V, which of the following is *not* true?

A) H is closed under addition B) K is closed under scalar multiplication. C) $0 \in H$ D) dim K $\leq$ dim V E) The intersection $H \cap K$ is also a subspace of V F) The union $H \cup K$ is also a subspace of V G) more than one of the above are untrue H) none of the above (all are true)

6. Let $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (in coordinates of the standard basis for $\mathbb{R}^2$) and let $\mathcal{B} = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}\}$ be an alternate basis. Find $[\mathbf{x}]_{\mathcal{B}}$.

A) $\begin{bmatrix} -1 \\ 11 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$ C) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} \frac{10}{3} \\ \frac{1}{3} \end{bmatrix}$

E) none of the above

Part II: True or False (30) Determine if each of the following statements is true or false. Each problem in this section is worth three points.

7. If $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix}$, then A is invertible.

   A) True B) False

8. Any subspace of a vector space is also a vector space.

   A) True B) False

9. A subset H of a vector space V is a subspace if the following are satisfied: (i) the zero vector of V is in H (ii) H is closed under vector addition, (iii) there exists a scalar c such that $c\mathbf{u}$ is in H whenever $\mathbf{u}$ is in H.

   A) True B) False

10. If A is invertible, the $Nul(A) = Nul(A^{-1})$.

    A) True B) False

11. There is a basis for $\mathbb{R}^4$ that *includes* the vectors $\begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \end{bmatrix}$

    A) True B) False

12. A null space is a vector space.

A) True B) False

13. Col(A) is the set of all solutions to the equation $A\mathbf{x} = \mathbf{b}$.

A) True B) False

14. If $S = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is a linearly independent subset of $\mathbb{R}^6$, then S is a basis for $\mathbb{R}^6$.

A) True B) False

15. The number of variables in the equation $A\mathbf{x} = \mathbf{b}$ equals the dimension Nul(A).

A) True B) False

16. The vectors $\begin{bmatrix} 1 \\ 10 \\ -1 \\ 7 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 0 \\ 3 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ span $\mathbb{R}^4$.

A) True B) False

Part III: Short Answer(20)

You do not need to show any work for this part, just answer each question or fill in the blank.

17. The matrix $A = \begin{bmatrix} 12 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$ reduces to $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) row(A) is a subspace of $\mathbb{R}^n$ where $n =$

(b) Col(A) is a subspace of $\mathbb{R}^n$ where $n =$

(c) Nul(A) is a subspace of $\mathbb{R}^n$ where $n =$

(d) rank(A)=

Give a basis for:

(e) row(A)

(f) Col(A)

(g) Nul(A)

Part IV: Longer Answer(20)

Show all appropriate work to receive full credit.

18. Clearly state whether each statement is true or false. If the statement is true, give a proof. If the statement is false, give a counterexample or a reason why it cannot be true.

    (a) If A and B are square $(n \times n)$ matrices, then

    $$det(AB) = det((AB)^{-1}).$$

    (b) Let $\mathcal{P}_2$ be the vector space of quadratic polynomials $p(t)$.
    Then $H = \{p(t) \in \mathcal{P}_2 | p(t) = at^2 + b, a, b \in \mathbb{R}\}$ is a subspace.

    (c) The map $D : M_{n\times n} \to \mathbb{R}$ defined by $D(A) = det(A)$ is a linear map.

19. For each matrix below, find the determinant of A.

(a) $A = \begin{bmatrix} 24 & -3 \\ 6 & 24 \end{bmatrix}$

(b) $A = \begin{bmatrix} 10 & -1 & 7 & 5 \\ 24 & 20 & 6 & 12 \\ 30 & -9 & 51 & 15 \\ 4 & -9 & 15 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} \frac{1}{2} & 1 & 3 \\ 10 & -1 & 5 \\ \frac{1}{3} & 0 & 0 \end{bmatrix}$