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Math 309 **PRACTICE** Final Exam

December 2016

You may use a scientific calculator, but no graphing calculators or other electronic devices allowed. You may use one hand-written (by you) 4×6 index card of notes. Fill out your scantron cards with your name (including your official "preferred" name) and your ID number. Note that each question, whether true/false or multiple choice, is worth 4 points.

Part I: True or False Determine if each of the following statements is true or false.

1. Every matrix is row equivalent to one and only one echelon matrix.

   A) True   B) False

2. The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable.

   A) True   B) False

3. A set of two vectors is linearly dependent if and only if one vector is a multiple of another.

   A) True   B) False

4. If a is a 3×5 matrix and T is a transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, then the domain of T is $\mathbb{R}^3$.

   A) True   B) False

5. The dimensions of the column space and the null space for an $m \times n$ matrix are equal.

   A) True   B) False

6. U is an $m \times n$ matrix with *orthonormal* columns if and only if $U^T U = I$.

   A) True   B) False

7. If U is an $m \times n$ matrix with *orthonormal* columns, and $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$, then $\|U\mathbf{x}\| = \|\mathbf{x}\|$.

   A) True   B) False

8. If $\mathbf{v}$ and $\mathbf{w}$ are eigenvectors of A for the same eigenvalue λ, then $\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

   A) True   B) False

9. If A is a matrix with real entries and $3 - 4i$ is an eigenvalue of A then $3 + 4i$ is also an eigenvalue.

   A) True   B) False

10. Every invertible matrix is diagonalizable.

    A) True   B) False

11. If $u \in W$, where W is a subspace of $\mathbb{R}^n$, the $proj_W \mathbf{u} = \mathbf{u}$.

    A) True   B) False

12. Suppose that A is a 6×6 matrix with characteristic equation $(\lambda - 3)(\lambda + 7)(\lambda^2 - 3)(\lambda^2 - 3\lambda - 1)$. Then A is diagonalizable.

    A) True   B) False

Part II: Multiple Choice Select the best answer for each question.

13. If A and B are 4×4 matrices, $\det(A) = 7$ and $\det(B) = 8$, what is $\det((2A)B^{-1})$?

    A) 1 B) -11 C) 14 D) 12 E) -12 F) 56 G) -112 H) none of the above

14. Consider the system of equations

$$5x_1 - 7x_2 = 12$$

$$25x_1 + hx_2 = k.$$

    Give conditions on h and k such that the system has *many* solutions.

    A) $h = -35$ B) $h = -35$ and $k = 60$ C)$h \neq -35$ and $k = 60$
    D)$h = -35$ and $k \neq 60$ E) $h \neq -35$ and $k \neq 60$ F) None of the above.

15. Let A be an $m \times n$ matrix and suppose the reduced echelon form of A has four rows of zeros. What is the rank of A?

    A) m B) n C) n - m D) n - 4 E) m - 4

    F) m + n G) m + n +2 H) none of the above

16. Let $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (in coordinates of the standard basis for $\mathbb{R}^2$) and let $\mathcal{B} = \{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}\}$ be an alternate basis. Find $[\mathbf{x}]_{\mathcal{B}}$.

    A) $\begin{bmatrix} -1 \\ 11 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$ C) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ D) $\begin{bmatrix} \frac{10}{3} \\ \frac{1}{3} \end{bmatrix}$

    E) none of the above

17. For a discrete dynamical system given by $\mathbf{x}_{k+1} = A\mathbf{x}_k$, the trajectories all spiral in towards the origin. Which of the following might be the eigenvalues of the 2×2 matrix A?

    A) 1, -2 B) 0.5, -2 C) 0.5, .75 D) 3, 4
    E) 0.9+0.2i, 0.9-0.2i F) 0.8+0.7i, 0.8-0.7i G) none of the above

18. Find a least squares solution to the inconsistent equation $Ax = b$ where $A = \begin{bmatrix} 1 & 5 \\ 2 & -2 \\ -1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$

A) $\begin{bmatrix} 17 \\ -12 \end{bmatrix}$ B) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$ C) $\begin{bmatrix} \frac{1}{3} \\ \frac{8}{15} \end{bmatrix}$ D) $\begin{bmatrix} \frac{1}{9} \\ \frac{1}{3} \end{bmatrix}$ E) $\begin{bmatrix} \frac{2}{3} \\ \frac{11}{13} \end{bmatrix}$

F) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$ G) none of the above

19. Let $V = P_2$, the vector space of degree two polynomials. Define an inner product on V by

$$f \cdot g = \int_0^1 f(x)g(x)dx.$$

In this inner product space are $p(x) = x + 4$ and $q(x) = x^2$ orthogonal?

A) Yes B) No

20. If $T : P_1 \to P_1$ is a transformation on linear functions such that $T(1 + x) = 3 + 2x$ and $T(2 - 3x) = 11 - x$, find $T(3 + 2x)$.

A) 75-30x B) 10+5x C) 12-3x D) 12+42x E) 12x F) None of the above

21. Let $V = P_2$, the vector space of degree two polynomials. Define an inner product on V for $f(x) = ax^2 + bx + c$ and $g(x) = rx^2 + sx + t$ by

$$f \cdot g = ar + bs + ct.$$

In this inner product space are $p(x) = x + 4$ and $q(x) = x^2$ orthogonal?

A) Yes B) No

22. Let $V = C^\infty(\mathbb{R})$, the vector space of smooth functions. By the Constant Multiplier and Sum rules for derivative, we know that the derivative is a linear transformation $D : V \to V$. Which of the following functions is **not** an eigenvector for D?

A) $f(x) = e^x$ B) $f(x) = e^{2x}$ C) $f(x) = 3e^x$
D) $f(x) = 3e^{2x}$ E) $f(x) = xe^{2x}$ F) $f(x) = 2^x$
G) none of the above

23. Which of the following conditions is *not* equivalent to an $n \times n$ matrix A being invertible?

A) The number 0 is an eigenvalue of A

B) $\det(A) \neq 0$

C) ColA=$\mathbb{R}^n$

D) rank$A$=n

E) dim Nul A=0

F) A^T is an invertible matrix

G) A has n pivot positions

H) The columns of A span $\mathbb{R}^n$

I) A is row equivalent to the identity matrix

J) The columns of A are linearly independent

K) none of the above

24. Which of the following vectors is *not* an eigenvector of $A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}$

A) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ D) $\begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$ E) $\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

F) All of the above are in the eigenspace

G)More than one of the above vectors is not in the eigenspace

25. The vectors $\mathbf{v} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$ are linearly independent and span a two dimensional subspace W of $\mathbb{R}^3$. Use the Gram-Schmidt process to find an orthogonal basis of W including $\mathbf{v}$. What is the other vector in the basis?

A) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$ C) $\begin{bmatrix} 5 \\ 4 \\ -7 \end{bmatrix}$ D) $\begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}$ E) $\begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$