Exam 1 Practice

Math 309 Fall 2016

October 5, 2016

No notes, calculators, or other electronic devices allowed.

Part I: Multiple Choice

1. Suppose A is an $n \times n$ matrix. Which of the following statements are NOT equivalent to A being invertible?

   (a) A^T is invertible

   (b) The solution to the system $Ax = 0$ has n free variables

   (c) The columns of A span $\mathbb{R}^n$

   (d) A has n pivots

   (e) The transformation $T(x) = Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

   A) a B) b C) d D) c E) e F) None of the above–all are equivalent.

2. Let C=BA where $A = \begin{bmatrix} 1 & -6 & 7 \\ 9 & 0 & 4 \\ 0 & 6 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{5}{6} & \frac{1}{6} & 3 \\ \frac{5}{6} & 2 & \frac{1}{6} \\ 1 & \frac{5}{6} & \frac{1}{6} \end{bmatrix}$. Find c_{32}.

   A) $\frac{15}{6}$ B) $\frac{83}{3}$ C) 0 D) 17 E) $\frac{103}{6}$ F) None of the above.

3. Consider the system of equations

$$x_1 - 5x_2 + 7x_3 = 12$$

$$3x_1 + hx_2 + 21x_3 = k.$$

   Give conditions on h and k such that the system is inconsistent.

   A) $h = 15$ and $k = 36$ B) $h \neq 15$ and $k = -36$ C)$h- = 15$ and $k = 36$ D)$h = -15$ and $k \neq -36$ E) $h = -15$ and $k \neq 36$ F) None of the above.

4. Which of the following transformations are linear?

   (a) $T(x_1) = (x_1, 0, 0, 0, 10x_1)$

   (b) $T(x_1, x_2) = (12x_1 + 2, x_2, x_1 + x_2)$

   (c) $T(x_1, x_2) = (x_1^2, x_2, x_1)$

   (d) $T(x_1, x_2, x_3) = (|x_1|, |x_2|, |x_3|)$

   (e) $T(x_1, x_2, x_3) = (0, x_2)$

   A) a and b B) a, b, and d C) c D) d E) a and e F) None of the above.

5. Given a transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, with associated matrix A, in which of the following situations is it possible that T might be onto?

   (a) The row-echelon form of A has a row of zeros.

   (b) Three rows in the row-echelon form of A do not have pivots.

   (c) $n > m$

   (d) $m > n$

   (e) The row-echelon form of A has a pivot in every column

   A) a B) b C) c D) d E) e F) None of the above G) More than one of the above

6. For a certain linear transformation T with associated matrix A, all of the following statements are true except for one. Determine the false statement.

   (a) The transformation is one-to-one.

   (b) The equation $T(x) = 0$ has non-trivial solutions

   (c) $T : \mathbb{R}^n \to \mathbb{R}^m$, where $n > m$

   (d) The columns of A are linearly dependent

   A) a B) b C) c D) d E) Based on the information, they might all be true

7. Which statement about matrices below is not true?

   (a) $(A^T)^T = A$

   (b) $(A^{-1})^{-1} = A$

   (c) $(AB)^T = A^T B^T$

   (d) $(AB)^{-1} = B^{-1}A^{-1}$

   A) a B) b C) c D) d E) Based on the information, they might all be true F) More than one is false

8. Determine the number of free variables for the system

$$x_1 + 3x_2 - 4x_3 + x_4 = 6$$

$$x_1 + 2x_2 + 3x_3 = 12.$$

   A) 0 B) 1 C) 2 D) 3 E) 4 F) None of the above

9. Consider a transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, with associated matrix A and B the row-echelon reduction of A. If T is onto, which of the following might be true?

   (a) $n = 4$, $m = 6$ and B has four pivots.

   (b) $n = 4$, $m = 6$ and B has six pivots.

   (c) $n = 6$, $m = 4$ and B has six pivots.

   (d) $n = 4$, $m = 6$ and B has five pivots.

   A) a B) b C) c D) d E) e F) None of the above

10. If $T : P_1 \to P_1$ is a transformation on linear functions such that $T(1 + 5x) = -4 + 4x$ and $T(4 + 19x) = 1 + 2x$, find $T(4 - 3x)$.

    A) 375-306x B) 42-9x C) 0 D) 12-42x E) 13x+1 F) None of the above

Part II: Hand graded

1. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the x_2 axis then reflects points through the line $x_2 = -x_1$.

   (a) Give the matrix A_1 for the transformation which reflects points through the x_2 axis.

   (b) Give the matrix A_2 for the transformation which reflects points through the line $x_2 = -x_1$.

   (c) Give the matrix A_T for the T described above.

   (d) Describe the relation between A_1, A_2, and A_T, showing any appropriate calculations supporting your claim.

2. Clearly state whether each statement is true or false. If the statement is true, give a proof. If the statement is false, give a counterexample or a reason why it cannot be true.

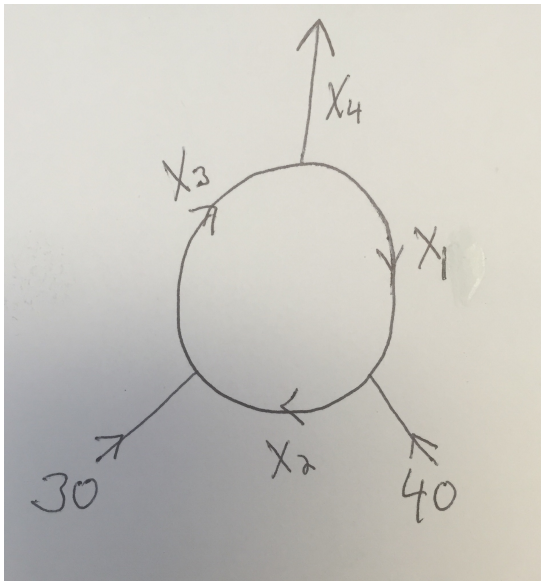
   (a) If A is an invertible matrix and $AC = AD$ then $C = D$.

   (b) If A and B are invertible $n \times n$ matrices, the

   $$(AB)^{-1} = A^{-1}B^{-1}.$$

   (c) Assuming the given matrices have sizes such that the given sums and products are defined, then

   $$A(B + C) = (B + C)A.$$



3. Find the general flow pattern for the network shown in the figure.

4. For each matrix below, either give a reason why it is not invertible or find the inverse.

(a) $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 2 & 6 & 6 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & -6 & 0 \\ 2 & -12 & 4 \\ 1 & -6 & 6 \end{bmatrix}$